Gravitation

Exercise

Q. 1. Study the entries in the following table and rewrite them putting the connected items in a single row.

I	II	III
Mass	m/s^2	Zero at the centre
Weight	kg	Measure of inertia
Acceleration due to gravity	Nm^2/kg^2	Same in the entire universe
Gravitational constant	N	Depends on height

Answer :

I	II	III	Explanation
Mass	Kg	Measure of inertia	Mass is the measure of an amount of matter present in the body of an object. And Inertia is the measure of the resistance of the body against an applied external force. Therefore Mass is the measure of inertia.
Weight	N	Depends on height	Weight is the force exerted on an object by earth. And S.I unit of force is N(Newton). Therefore S.I unit of weight is also N. Since W = mg and value of 'g' changes with the height therefore the weight also changes with the height.

Acceleration due to gravity	m/s^2	Zero at the centre	Acceleration due to gravity is basically the acceleration caused by gravity. And S.I unit of acceleration is m/s^2 therefore its S.I unit is also m/s^2. Since $g = \frac{GM}{R^2}$ and due to decrease in value of 'M' and 'R' as we go inside the earth the value of 'g' becomes aero at center.
Gravitational constant	Nm^2/kg^2	Same in the entire universe	Since $F = \frac{Gm_1m_2}{R^2}$ $\Rightarrow G = \frac{FR^2}{m_1m_2}$ And S.I unit of F is N, m_1,m_2 is kg, R is m. Therefore S.I unit of G = Nm^2/kg^2. And it is constant therefore same in entire universe.

Q. 2. A. Answer the following question.

What is the difference between mass and weight of an object. Will the mass and weight of an object on the earth be same as their values on Mars? Why?

Answer : Following are the difference between the weight and mass of an object.

MASS	WEIGHT
1.) Mass is the amount of matter present in the object	Weight of an object is the force with which an earth attracts the object.
2.) S.I unit of mass is Kg	S.I unit of weight is Newton(N).
3.) Its value is same everywhere.	Weight of an object changes from place to place.
4.) Weighing balance is used to measure mass of an object.	Spring balance is used to measure weight of an object.
5.) Mass of an object can never be zero	Weight of an object can be zero.

i. Since mass is the amount of the matter present in the body, and amount of matter (ex: bones, blood, skin etc) is same everywhere. Therefore the mass of the object on earth will be same as that on mars.

ii. Whereas the weight of an object on Earth and Mars will be different as accelearation due to gravity (g) is different for Earth and Mars and we know that **Weight= Mass × g**

Q. 2. B. Answer the following question.

What are (i) free fall, (ii) acceleration due to gravity (iii) escape velocity (iv) centripetal force?

Answer : (i) Free Fall: Whenever an object moves under the influence of the force of gravity alone, it is said to be in Free Fall. During free fall initial velocity of object is zero and force of air also acts on an object. Thus real free fall is possible only in vacuum because there is no air.

For Example:

a) When an object is dropped from the top of the table it falls down only due to gravitational force hence it is under free fall.

b) An apple falling from the tree.

But when we are standing on the ground, flying in a flight we are not in free fall because other forces except gravitational force are also acting.

(ii) Acceleration due to gravity: The acceleration which is gained by an object because of the gravitational force is called acceleration due to gravity. Its S.I unit is m/s^2. It is denoted by 'g' and its value at surface of earth is $9.8 m/s^2$.

It is a vector quantity (have both magnitude and direction) and it is directed towards the center of the earth. The value of acceleration due to gravity is not fixed it changes from place to place Like at moon 'g' value is one-sixth of value on earth.

(iii) Escape velocity: The minimum value of initial velocity at which an object to escape from the gravitational pull/force of the earth and never comes back to the earth is called as the escape velocity.

The escape velocity from earth is about 11.186 km/s, means if an object travels 11.186 km in 1 sec it will escape from earth's gravitational pull. All the satellites and rockets are launched with velocity equal to escape velocity in order to escape from the earth's gravity.

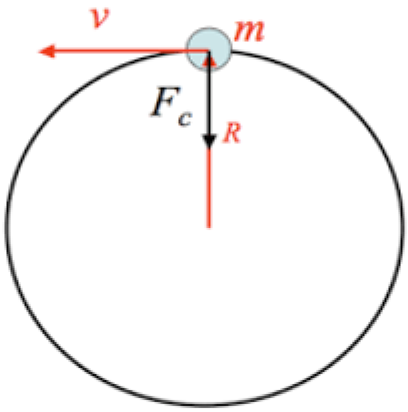
(iv) Centripetal force: The force that acts on an object to keep it moving along the circular path with constant speed is called as the centripetal force.

It is given by $F_c = \frac{mv^2}{r}$

It is directed towards the center of the circle (of radius 'r') in which the object is moving (with velocity 'v').

Example: The stone tied to a piece of string whirl in circle due to centripetal force only.

The diagram below illustrates the centripetal force acting on an object towards the center.

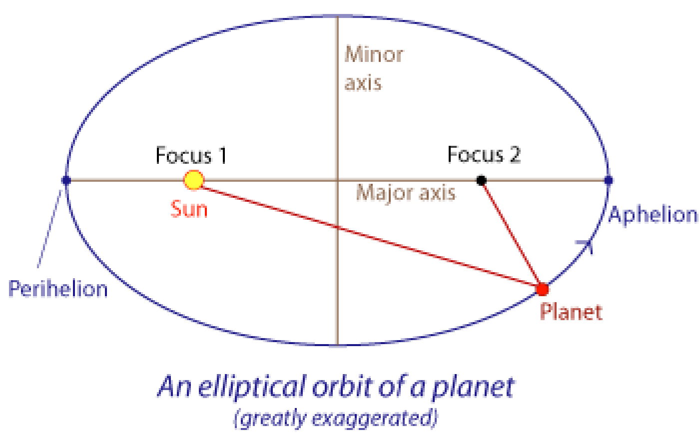


Q. 2. C. Answer the following question.

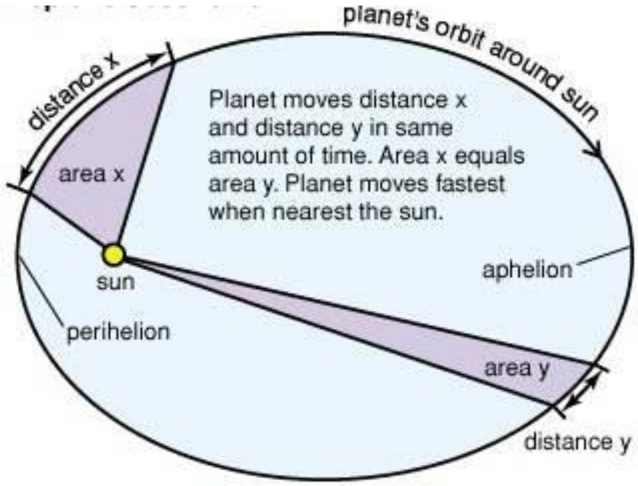
Write the three laws given by Kepler. How did they help Newton to arrive at the inverse square law of gravity?

Answer : Johannes Kepler studied about the planets motion and their positions and. He noticed that motion of the planets follows a certain law. He gave three laws describing the planetary motion. These are known as Kepler's laws which are as follows:

Kepler's First law: The orbit of planet is an ellipse with sun at one of the foci. The figure below illustrates the elliptical orbit of earth with sun at its focus.



Kepler's Second law: The line joining the planet and the Sun sweeps equal areas in equal intervals of time.



In the above figure $A_1 = A_2$ according to law.

Kepler's Third law: The Square of its period of revolution around the Sun is directly proportional to the cube of the mean distance of a planet from the Sun.

If 'r' is the average distance of the planet from the sun and 'T' is the period of revolution then according to third law;

$T^2 \propto r^3$ i.e

$$\frac{T^2}{r^3} = \text{constant} = K$$

Inverse Square law of gravity: We know that centripetal force F is given by ;

$F = \frac{mv^2}{r}$

Where v = velocity of the planet = $\frac{\text{Distance travelled}}{\text{timetaken}}$

Distance travelled in one revolution=$2\pi r$

Where r = radius of the orbit.

$V = \frac{2\pi r}{T}$

$F = \frac{mv^2}{r}$

$$= \frac{m\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$= \frac{4m\pi^2 r^2}{rT^2}$$

$$= \frac{4m\pi^2 r}{T^2}$$

According to Kepler's third law;

$$\frac{T^2}{r^3} = \text{constant} = K$$

Hence

F = $\frac{4m\pi^2 r^3}{T^2 r^2}$ (Multiplying and dividing by r^2)

F = $\frac{4m\pi^2}{\left(\frac{T^2}{r^3}\right) r^2}$

F = $\frac{4m\pi^2}{Kr^2}$ (By Kepler's third law).

But $\frac{4m\pi^2}{K}$ is constant

Therefore $F \propto \frac{1}{r^2}$

And this is <u>the newton's inverse square law which states that the centripetal force acting on planet is inversely proportional to the square of the distance ('r') between the sun and the planet</u>

Q. 2. D. Answer the following question.

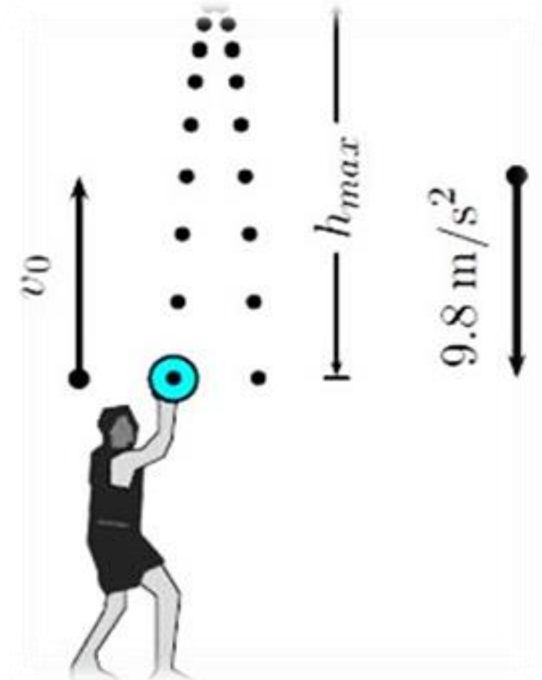
A stone thrown vertically upwards with initial velocity u reaches a height 'h' before coming down. Show that the time taken to go up is same as the time taken to come down.

Answer : Given;

Initial velocity = u;

Distance travelled(s) = h;

Diagram below shows the situation given in the question;



Time to go up (t_1): By Newton's first equation of the motion

v = u + at

Where v = final velocity;

u = initial velocity

a = acceleration;

t = time taken;

According to our question;

v = 0 (because object has reach a maximum height 'h' before coming down and velocity at maximum height is zero);

u= u;

a = -g (because when object will be going up the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is opposite therefore a is negative);

T = t_1;

Putting the values in the equation of motion we get;

$$0 = u + (-gt_1)$$

$$0 = u - gt_1.$$

$$u = gt_1$$

$$\Rightarrow t_1 = \frac{u}{g}$$

<u>Time of descent (t_2)</u>: By Newton's second equation of motion

$$s = ut + \frac{1}{2}at^2$$

Where;

s = Distance travelled;

u = initial velocity;

a = acceleration;

t = time taken;

According to our question;

s = h;

u = 0 (When object is at maximum height its velocity is zero);

a = g (because when object will be down the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is same therefore a is positive);

T = t_2;

Putting the values in the equation we get;

$$h = 0 + \frac{1}{2} g t_2^2.$$

$$h = \frac{g t_2^2}{2}$$

From newton's third equation of the motion;

$$v^2 = u^2 + 2as$$

Where symbols have usual meanings as above;

When object is descending down

u = 0 (When object is at maximum height its velocity is zero);

v = u (Final velocity will be same as initial velocity);

a = g (because when object will be down the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is same therefore a is positive);

$$s = \frac{g t_2^2}{2};$$

Putting values in the equation of motion;

Hence

$$u^2 = 2g \times g \times \frac{t_2^2}{2}$$

$$u^2 = g^2 \times t_2^2 \;\Rightarrow\; t_2^2 = \frac{u^2}{g^2};$$

$$t_2 = \frac{u}{g}$$

$$\text{Since } t_1 = t_2 = \frac{u}{g}$$

Hence time of ascent and time of descent are equal;

Q. 2. E. Answer the following question.

If the value of g suddenly becomes twice its value, it will become two times more difficult to pull a heavy object along the floor. Why?

Answer : For lifting anybody of mass ‘m’ the force(F_1) required is equal to ‘mg’ where

g = acceleration due to gravity;

(Since Force= Mass × Accleration

Therefore F_1 = mg

Now if the value of ‘g’ suddenly becomes twice its value than force(F_2) required will be

$F_2 = m2g = 2mg.$

We can see that $\frac{F_2}{F_1} = 2$

Therefore it becomes difficult to pull a heavy object if value of ‘g’ becomes twice its value as double the force is required to overcome the acceleration produced by the force of attraction of earth acting in downward direction

Q. 3. Explain why the value of g is zero at the centre of the earth.

Answer : The value of ‘g’ changes as we go inside the earth. According to formula of g = $\frac{GM}{R_2}$. As we go deep inside the earth the Value of ’R’ decreases therefore according to formula the value of ‘g’ should increase. But it is not so because as we go inside the earth towards its center the part of the earth(M) responsible for the gravitational force also decreases that is the value of the ‘M’ also decreases.

And due to the combined result of decrease of ‘R’ and ‘M’ the value of ‘g’ decreases as we go inside the earth and is zero at the center of the earth.

Q. 4. Let the period of revolution of a planet at a distance R from a star be T. Prove that if it was at a distance of 2R from the star, its period of revolution will be √8 T.

Answer : By Kepler’s Third law of planetary motion;

$$T^2 \propto R^3 \text{ i.e } \frac{T^2}{R^3} = \text{constant} = K$$

Where 'r' is the distance of planet from the star; 'T' is the period of revolution.

Let T_1 be the time period when planet was at a distance of '2R' from the star.

By Kepler's third law;

$$\frac{T_1^2}{(2R)^3} = K \Rightarrow K = \frac{T_1^2}{8R^3}$$

Since K is constant and its value is same therefore;

$$\frac{T^2}{R^3} = \frac{T_1^2}{8R^3}$$

$$\Rightarrow \frac{T_1^2}{T^2} = \frac{8R^3}{R^3}$$

$$\Rightarrow \frac{T_1^2}{T^2} = 8$$

$$\Rightarrow \frac{T_1}{T} = \sqrt{8}$$

$$\Rightarrow T_1 = \sqrt{8}\,T$$

<u>Hence Period of revolution will be $\sqrt{8}$ times the period of revolution when planet was at distance 'R' from the star.</u>

Q. 5. A. Solve the following example.

An object takes 5 s to reach the ground from a height of 5 m on a planet. What is the value of g on the planet?

Answer : Given;

Time for object to reach ground (t) = 5 s;

Height of object from the ground(s) = 5m;

Acceleration due to gravity (g) =? ;

Since when Object is released from the height of 5m it will be in free fall hence its initial velocity (u) = 0;

Now, By Newton's second equation of motion

$$s = ut + \frac{1}{2}at^2$$

Here u = 0, a = g, t = 5s, s = 5m;

Putting these values in the above equation we get;

$$5 = 0 \times t + \frac{1}{2}g \times 5^2$$

$\Rightarrow 5 = \frac{1}{2}(g \times 25)$.

$\Rightarrow 10 = g \times 25$.

$\Rightarrow g = \frac{10}{25} = 0.4 m/s^2$.

Hence acceleration due to gravity on the planet is $0.4 m/s^2$.

Q. 5. B. Solve the following example.
The radius of planet A is half the radius of planet B. If the mass of A is M_A, what must be the mass of B so that the value of g on B is half that of its value on A?

Answer : We know that acceleration due to gravity (g) is given by

$$g = \frac{GM}{R^2}$$

Where G = universal gravitational constant;

M = mass of the planet;

R = radius of the planet;

Mass of Planet A = M_A;

Radius of Planet A = R_A;

Value of ‘g’ on A = g_A;

$$g_A = \frac{GM_A}{R_A^2}$$

Mass of Planet B = M_B;

Radius of Planet B = R_B;

Value of ‘g’ on B = g_B;

$$g_B = \frac{GM_B}{R_B^2}$$

Also it is given that;

$$R_A = \frac{R_B}{2}$$

$$\Rightarrow \frac{R_B}{R_A} = 2$$

$$g_B = \frac{g_A}{2}$$

$$\Rightarrow \frac{g_A}{g_B} = 2$$

Therefore;

$$\frac{g_A}{g_B} = \frac{\frac{GM_A}{R_A^2}}{\frac{GM_B}{R_B^2}}$$

$$\frac{g_A}{g_B} = \left(\frac{GM_A}{R_A^2}\right) \times \left(\frac{R_B^2}{GM_B}\right)$$

$$\frac{g_A}{g_B} = \left(\frac{M_A}{M_B}\right) \times \left(\frac{R_B^2}{R_A^2}\right)$$

$$\frac{g_A}{g_B} = \left(\frac{M_A}{M_B}\right) \times \left(\frac{R_B}{R_A}\right)^2$$

$$2 = \left(\frac{M_A}{M_B}\right) \times (2)^2 \Rightarrow 2 = \left(\frac{M_A}{M_B}\right) \times 4$$

(From question $\frac{R_B}{R_A} = 2$ and $\frac{g_A}{g_B} = 2$)

$\Rightarrow \frac{M_A}{M_B} = \frac{1}{2} \Rightarrow M_B = 2M_A$

<u>Therefore the mass of planet B should be two times the mass of planet A i.e $M_B = 2M_A$</u> .

Q. 5. C. Solve the following example.

The mass and weight of an object on earth are 5 kg and 49 N respectively. What will be their values on the moon? Assume that the acceleration due to gravity on the moon is 1/6th of that on the earth.

Answer : **<u>Mass value:</u>** Since mass is the amount of matter present in our body and it remains same irrespective of the change in position. Therefore the value of mass will be 5 Kg on moon as well.

<u>Weight value:</u> Weight is the force with which an object is attracted by a planet and it is equal to;

$$\text{Weight}(W) = F = mg$$

Where m = mass of the object;

g = acceleration due to gravity;

On earth weight(W_1) = mg = 49N;

On moon weight(W_2) = $\frac{mg}{6}$ (acceleration due to gravity is one-sixth of earth).

Hence weight on moon = $\frac{mg}{6} = \frac{49}{6} = 8.17N$

<u>Therefore Mass on moon is 5kg and weight on moon is 8.17N.</u>

Q. 5. D. Solve the following example.

An object thrown vertically upwards reaches a height of 500 m. What was its initial velocity? How long will the object take to come back to the earth? Assume g = 10 m/s^2

Answer : From newton's third equation of the motion;

$$v^2 = u^2 + 2as$$

Where;

V = Final velocity;

U = initial velocity;

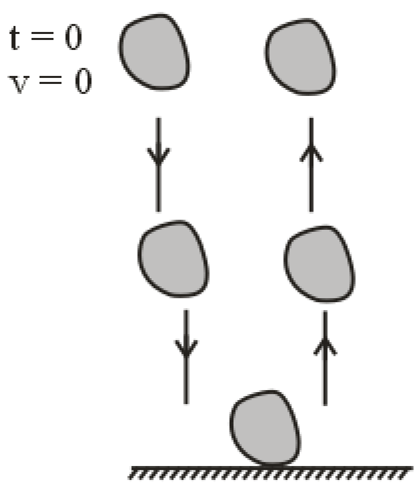
T = time taken;

S = distance travelled;

A = acceleration;

According to our question;

The figure below illustrates the situation given in the question



V = 0 (Velocity at maximum height is zero);

S = 500m;

A = -10m/s^2 (because when object will be going up the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is opposite therefore a is negative);

Putting the above values we get

$$0^2 = u^2 + (2 \times (-10) \times 500)$$

$$0 = u^2 - 1000$$

$$u^2 = 1000$$

$\Rightarrow$u= $\sqrt{}$1000=100

<u>Therefore initial velocity is 100m/s</u>.

From the Newton's first law of motion;

$$v = u + at$$

Where symbols have there usual meanings as above;

v = 0 (velocity at maximum height is zero);

u = initial velocity = 100m/s;

a = - 10m/s^2 (because when object will be going up the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is opposite therefore a is negative);

Putting the values we get;

$$0 = 100 + (-10t);$$

$$\Rightarrow 100 = 10t$$

$$t = \frac{100}{10} = 10s$$

Now we know that time required by an object to go up is same as time required to come down.

Therefore;

Total time = time of ascent + time of descent = 10 + 10 = 20 s

Hence total time to come back to earth is 20 seconds.

Q. 5. E. Solve the following example.

A ball falls off a table and reaches the ground in 1 s. Assuming g = 10 m/s^2, calculate its speed on reaching the ground and the height of the table.

Answer : From the Newton's first law of motion;

$$v = u + at$$

V = Final velocity;

U = initial velocity;

T = time taken;

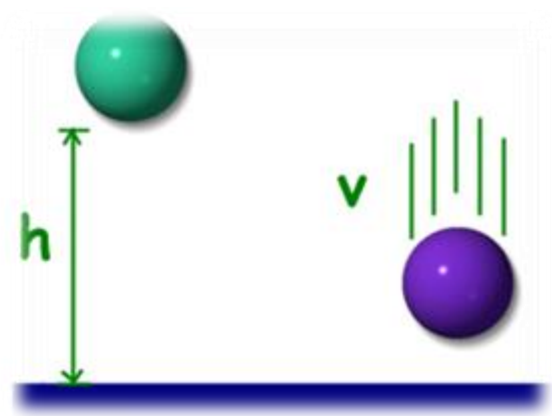
A = acceleration;

According to our question;

u = 0 (ball is at table and is falling from it (free fall) hence its initial velocity is zero);

A = 10m/s^2 (because when object will be down the acceleration due to gravity will be acting downwards to make object to fall. Hence by sign convention direction of motion and acceleration is same therefore a is positive);
T = 1 sec (given);

The figure below describes the ball falling from the table on ground with final velocity v.



Putting the values we get;

$$v = 0 + 10 \times 1;$$

$$v = 10 \times 1 \Rightarrow v = 10$$

Hence final velocity on reaching the ground is 10m/s.

Height of table Now, By Newton's second equation of motion

$$s = ut + \frac{1}{2}at^2$$

According to our question;

s = Table height;

u = 0 (ball is at table (maximum height) and is falling from it hence its initial velocity is zero);

a = 10m/s^2.

t = 1s.

Putting the above values we get

$$s = 0 + \frac{1}{2} \times 10 \times 1^2$$

$$s = \frac{1}{2} \times 10 \times 1 \Rightarrow s = \frac{10}{2} = 5$$

Hence height of the table is 5m

Q. 5. F. Solve the following example.

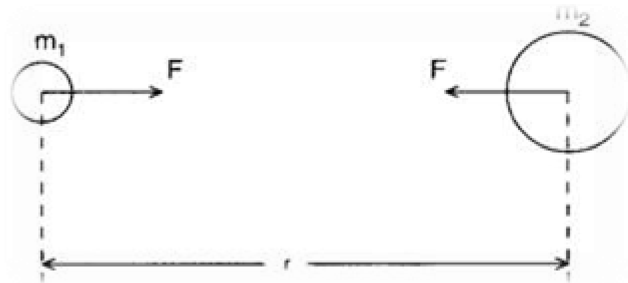
The masses of the earth and moon are 6 x 10^{24} kg and 7.4x10^{22} kg, respectively. The distance between them is 3.84 x 10^5 km. Calculate the gravitational force of attraction between the two?
Use G = 6.7 x 10^{-11} N m^2 kg^{-2}

Answer : Gravitational force of attraction between two bodies of masses m_1 and $m_2$separated by a distance of R is given by;

$$F = \frac{Gm_1m_2}{R^2}$$

G = Gravitational constant;

Figure below illustrates the question well;



Putting values of $m_1 = 6 \times 10^{24}$kg;

$m_2 = 7.4 \times 10^{22}$kg;

$R = 3.84 \times 10^5$km $= 3.84 \times 10^8$m (since 1km = 1000m);

$$F = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24} \times 7.4 \times 10^{22}}{(3.84 \times 10^8)^2}$$

$$F = \frac{6.7 \times 7.4 \times 6 \times 10^{35}}{3.84 \times 3.84 \times 10^{16}}$$

$$F = \frac{297.48 \times 10^{35}}{14.7456 \times 10^{16}}$$

$$F = \frac{297.48 \times 10^{19}}{14.7456}$$

$$F = 20.17 \times 10^{19} = 2.0 \times 10^{20}\ \mathrm{N}$$

Therefore the gravitational force of attraction between the earth and the moon is 2.0×10^{20} N.

Q. 5. G. Solve the following example.

The mass of the earth is 6 x 10^{24} kg. The distance between the earth and the Sun is 1.5x 10^{11} m. If the gravitational force between the two is 3.5 x 10^{22} N, what is the mass of the Sun?
Use G = 6.7 x 10^{-11} N m^2 kg^{-2}

Answer : Gravitational force of attraction between two bodies of masses m_1 and $m_2$separated by a distance of R is given by;

$$F = \frac{Gm_1 m_2}{R^2}$$

G = Gravitational constant

Putting values of m_1 = $6 \times 10^{24} kg$;

$m_2 = ?$;

$R = 1.5 \times 10^{11}$ m.

$F = 3.5 \times 10^{22} N$

$$F = 3.5 \times 10^{22} = \frac{6.7 \text{ x } 10^{-11} \times 6 \times 10^{24} \times m_2}{(1.5 \times 10^{11})^2}$$

$$\Rightarrow 3.5 \times 10^{22} = \frac{6.7 \text{ x } 6 \times 10^{13} \times m_2}{2.25 \times 10^{22}}$$

$$m_2 = \frac{3.5 \times 2.25 \times 10^{22} \times 10^{22}}{6.7 \times 6 \times 10^{13}}$$

Since $x^a \times x^b = x^{a+b}$

$$m_2 = \frac{7.875 \times 10^{44}}{40.2 \times 10^{13}}$$

Since $\frac{x^a}{x^b} = x^{a-b}$

$$m_2 = 0.195895 \times 10^{31}$$

$m_2 = 1.96 \times 10^{30}$ Kg.

Therefore mass of the sun is $\underline{1.96 \times 10^{30} \text{ Kg}}$