Arithmetic Progression

Practice Set 3.1

Q. 1 A. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

2, 4, 6, 8, . . .

Answer :

2, 4, 6, 8, . . .

Here, the first term, $a_1 = 2$

Second term, $a_2 = 4$

$a_3 = 6$

Now, common difference = $a_2 - a_1 = 4 - 2 = 2$

Also, $a_3 - a_2 = 6 - 4 = 2$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = 2.

Q. 1 B. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

$$2, \frac{5}{2}, 3, \frac{7}{3}, \ldots$$

Answer :

$$2, \frac{5}{2}, 3, \frac{7}{3}, \ldots..$$

Here, the first term, $a_1 = 2$

Second term, $a_2 = \frac{5}{2}$

Third Term, $a_3 = 3$

Now, common difference = $a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

Also, $a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, $d = \frac{1}{2}$.

Q. 1 C. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

– 10, – 6, – 2, 2, . . .

Answer :

– 10, – 6, – 2,2, . . .

Here, the first term, $a_1 = -10$

Second term, $a_2 = -6$

$a_3 = -2$

Now, common difference = $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

Also, $a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = 4.

Q. 1 D. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

0.3, 0.33, .0333, . . .

Answer :

0.3, 0.33, 0.333,…..

Here, the first term, $a_1 = 0.3$

Second term, $a_2 = 0.33$

$a_3 = 0.333$

Now, common difference = $a_2 – a_1 = 0.33 – 0.3 = 0.03$

Also, $a_3 – a_2 = 0.333 – 0.33 = 0.003$

Since, the common difference is not same.

Hence the terms are not in Arithmetic progression

Q. 1 E. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

0, – 4, – 8, – 12, . . .

Answer :

0, – 4, – 8, – 12, . . .

Here, the first term, $a_1 = 0$

Second term, $a_2 = – 4$

$a_3 = – 8$

Now, common difference = $a_2 – a_1 = – 4 – 0 = – 4$

Also, $a_3 – a_2 = – 8 – (– 4) = – 8 + 4 = – 4$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = – 4.

Q. 1 F. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

$$-\frac{1}{5},-\frac{1}{5},-\frac{1}{5},...$$

Answer :

$$-\frac{1}{5},-\frac{1}{5},-\frac{1}{5},....$$

Here, the first term, $a_1 = -\frac{1}{5}$

Second term, $a_2 = -\frac{1}{5}$

$a_3 = -\frac{1}{5}$

Now, common difference $= a_2 - a_1 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

Also, $= a_3 - a_2 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, $d = 0$.

Q. 1 G. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

$$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2},...$$

Answer :

$3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, ...$

Here, the first term, $a_1 = 3$

Second term, $a_2 = 3 + \sqrt{2}$

$a_3 = 3 + 2\sqrt{2}$

Now, common difference = $a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

Also, $a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, $d = \sqrt{2}$.

Q. 1 H. Which of the following sequences are A.P. ? If they are A.P. find the common difference.

127, 132, 137, . . .

Answer : 127, 132, 137, . . .

Here, the first term, $a_1 = 127$

Second term, $a_2 = 132$

$a_3 = 137$

Now, common difference = $a_2 - a_1 = 132 - 127 = 5$

Also, $a_3 - a_2 = 137 - 132 = 5$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = 5.

Q. 2 A. Write an A.P. whose first term is a and common difference is d in each of the following.

a = 10, d = 5

Answer : a = 10, d = 5

Let $a_1 = a = 10$

Since, the common difference d = 5

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = 10 + 5 = 15$

$a_3 = a_2 + d = 15 + 5 = 20$

$a_4 = a_3 + d = 20 + 5 = 25$

Hence, An A.P with common difference 5 is 10, 15, 20, 25,….

Q. 2 B. Write an A.P. whose first term is a and common difference is d in each of the following.

a = – 3, d = 0

Answer :

a = – 3, d = 0

Let $a_1 = a = -3$

Since, the common difference d = 0

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -3 + 0 = -3$

$a_3 = a_2 + d = -3 + 0 = -3$

$a_4 = a_3 + d = -3 + 0 = -3$

Hence, An A.P with common difference 0 is – 3, – 3, – 3, – 3,….

Q. 2 C. Write an A.P. whose first term is a and common difference is d in each of the following.

$$a = -7, d = \frac{1}{2}$$

Answer :

$a = -7,\ d = \frac{1}{2}$

Let $a_1 = a = -7$

Since, the common difference $d = \frac{1}{2}$

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -7 + \frac{1}{2} = \frac{-14+1}{2} = -\frac{13}{2}$

$$a_3 = a_2 + d = -\frac{13}{2} + \frac{1}{2} = \frac{-13 + 1}{2} = -\frac{12}{2} = -6$$

$$a_4 = a_3 + d = -6 + \frac{1}{2} = \frac{-12 + 1}{2} = -\frac{11}{2}$$

Hence, An A.P with common difference $\frac{1}{2}$ is $-7, -\frac{13}{2}, -6, -\frac{11}{2}, \ldots..$

Q. 2 D. Write an A.P. whose first term is a and common difference is d in each of the following.

a = – 1.25, d = 3

Answer : a = – 1.25, d = 3

Let a_1 = a = – 1.25

Since, the common difference d = 3

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -1.25 + 3 = 1.75$

$a_3 = a_2 + d = 1.75 + 3 = 4.75$

$a_4 = a_3 + d = 4.75 + 3 = 7.75$

Hence, An A.P with common difference 3 is – 1.25, 1.75, 4.75, 7.75

Q. 2 E. Write an A.P. whose first term is a and common difference is d in each of the following.

a = 6, d = – 3

Answer :

$a = 6, d = -3$

Let $a_1 = a = 6$

Since, the common difference $d = -3$

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = 6 + (-3) = 6 - 3 = 3$

$a_3 = a_2 + d = 3 + (-3) = 3 - 3 = 0$

$a_4 = a_3 + d = 0 + (-3) = -3$

Hence, An A.P with common difference – 3 is 6, 3, 0, – 3…

Q. 2 F. Write an A.P. whose first term is a and common difference is d in each of the following.

a = – 19, d = – 4

Answer :

$a = -19, d = -4$

Let $a_1 = a = -19$

Since, the common difference $d = -4$

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -19 + (-4) = -19 - 4 = -23$

$a_3 = a_2 + d = -23 + (-4) = -23 - 4 = -27$

$a_4 = a_3 + d = -27 + (-4) = -27 - 4 = -31$

Hence, An A.P with common difference – 4 is – 19, – 23, – 27, – 31,….

Q. 3 A. Find the first term and common difference for each of the A.P.

5, 1, – 3, – 7, . . .

Answer :

5, 1, – 3, – 7, . . .

First term $a_1 = 5$

Second term $a_2 = 1$

Third term $a_3 = -3$

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = 1 - 5 = -4$

Hence, the common difference $d = -4$ and first term is 5

Q. 3 B. Find the first term and common difference for each of the A.P.

0.6, 0.9, 1.2, 1.5, . . .

Answer :

0.6, 0.9, 1.2, 1.5, . . .

First term $a_1 = 0.6$

Second term $a_2 = 0.9$

Third term $a_3 = 1.2$

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = 0.9 - 0.6 = 0.3$

Hence, the common difference $d = 0.3$ and first term is 0.6

Q. 3 C. Find the first term and common difference for each of the A.P.

127, 135, 143, 151, . . .

Answer : 127, 135, 143, 151, . . .

First term $a_1 = 127$

Second term $a_2 = 135$

Third term $a_3 = 143$

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = 135 - 127 = 8$

Hence, the common difference d = 8 and first term is 127

Q. 3 D. Find the first term and common difference for each of the A.P.

$$\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$$

Answer :

$$\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots.$$

First term $a_1 = \frac{1}{4}$

Second term $a_2 = \frac{3}{4}$

Third term $a_3 = \frac{5}{4}$

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the common difference $d = \frac{1}{2}$ and first term is $\frac{1}{4}$

Practice Set 3.2

Q. 1 A. Write the correct number in the given boxes from the following A. P.

1, 8, 15, 22, . . .

Here

$$a = \square, t_1 = \square, t_2 = \square, t_3 = \square,$$
$$t_2 - t_1 = \square - \square = \square$$
$$t_3 - t_2 = \square - \square = \square \quad \therefore d = \square$$

Answer :

1, 8, 15, 22, . . .

First term a = 1

Second term t_1 = 8

Third term t_2 = 15

Fourth term t_3 = 22

We know that $d = t_{n+1} - t_n$

Thus, $t_2 - t_1 = 15 - 8 = 7$

$t_3 - t_2 = 22 - 15 = 7$

Thus, d = 7

Q. 1 B. Write the correct number in the given boxes from the following A. P.

3, 6, 9, 12, . . .

$$\text{Here } t_1 = \square, t_2 = \square, t_3 = \square, t_4 = \square,$$
$$t_2 - t_1 = \square, t_3 - t_2 = \square$$
$$\therefore d = \square$$

Answer : 3,6,9,12, . . .

First term a = 3

Second term t_1 = 6

Third term t_2 = 9

Fourth term $t_3 = 12$

We know that $d = t_{n+1} - t_n$

Thus, $t_2 - t_1 = 9 - 6 = 3$

$t_3 - t_2 = 12 - 9 = 3$

Thus, $d = 3$

Q. 1 C. Write the correct number in the given boxes from the following A. P.

– 3, – 8, – 13, – 18, . . .

Here $t_3 = \square, t_2 = \square, t_4 = \square, t_1 = \square,$

$t_2 - t_1 = \square, t_3 - t_2 = \square$

$\therefore \ a = \square, d = \square$

Answer : – 3, – 8, – 13, – 18, . . .

First term $a = -3$

Second term $t_1 = -8$

Third term $t_2 = -13$

Fourth term $t_3 = -18$

We know that $d = t_{n+1} - t_n$

Thus, $t_2 - t_1 = -13 - (-8) = -13 + 8 = -5$

$t_3 - t_2 = -18 - (-13) = -18 + 13 = -5$

Thus, $d = -5$

Q. 1 D. Write the correct number in the given boxes from the following A. P.

70, 60, 50, 40, . . .

Here $t_1 = \square, t_2 = \square, t_3 = \square, \ldots$

$\therefore \; a = \square, d = \square$

Answer : 70, 60, 50, 40, . . .

First term a = 70

Second term $t_1 = 60$

Third term $t_2 = 50$

Fourth term $t_3 = 40$

We know that $d = t_{n+1} – t_n$

Thus, $t_2 – t_1 = 50 – 60 = – 10$

$t_3 – t_2 = 40 – 50 = – 10$

Thus, d = – 10

Q. 2. Decide whether following sequence is an A.P., if so find the 20th term of the progression.

– 12, – 5, 2, 9, 16, 23, 30, . . .

Answer : Given A.P. is – 12, – 5, 2, 9, 16, 23, 30, . . .

Where first term a = – 12

Second term $t_1 = – 5$

Third term $t_2 = 2$

Common Difference $d = t_2 – t_1 = 2 – (– 5) = 2 + 5 = 7$

We know that, n^{th} term of an A.P. is

$t_n = a + (n – 1)d$

We need to find the 20^{th} term,

Here n = 20

Thus, $t_{20} = -12 + (20 - 1) \times 7$

$t_{20} = -12 + (19) \times 7 = -12 + 133 = 121$

Thus, $t_{20} = 121$

Q. 3. Given Arithmetic Progression 12, 16, 20, 24, . . . Find the 24th term of this progression.

Answer : Given A.P. is 12, 16, 20, 24, . . .

Where first term a = 12

Second term $t_1 = 16$

Third term $t_2 = 20$

Common Difference $d = t_2 - t_1 = 20 - 16 = 4$

We know that, n^{th} term of an A.P. is

$t_n = a + (n - 1)d$

We need to find the 24^{th} term,

Here n = 24

Thus, $t_{24} = 12 + (24 - 1) \times 4$

$t_{24} = 12 + (23) \times 4 = 12 + 92 = 104$

Thus, $t_{24} = 104$

Q. 4. Find the 19th term of the following A.P.

7, 13, 19, 25, . . .

Answer : Given A.P. is 7, 13, 19, 25, . . .

Where first term a = 7

Second term $t_1 = 13$

Third term $t_2 = 19$

Common Difference $d = t_2 – t_1 = 19 – 13 = 6$

We know that, n^{th} term of an A.P. is

$t_n = a + (n – 1)d$

We need to find the 19^{th} term,

Here n = 19

Thus, $t_{19} = 7 + (19 – 1)\times 6$

$t_{19} = 7 + (18)\times 6 = 7 + 108 = 115$

Thus, $t_{19} = 115$

Q. 5. Find the 27th term of the following A.P.

9, 4, – 1, – 6, – 11, . . .

Answer : Given A.P. is 9, 4, – 1, – 6, – 11, . . .

Where first term a = 9

Second term $t_1 = 4$

Third term $t_2 = – 1$

Common Difference $d = t_2 – t_1 = – 1 – 4 = – 5$

We know that, n^{th} term of an A.P. is

$t_n = a + (n – 1)d$

We need to find the 27^{th} term,

Here n = 27

Thus, $t_{27} = 9 + (27 – 1)\times (– 5)$

$t_{27} = 9 + (26)\times (– 5) = 9 – 130 = – 121$

Thus, $t_{27} = – 121$

Q. 6. Find how many three digit natural numbers are divisible by 5.

Answer : List of three digit number divisible by 5 are

100, 105,110,115,……….. 995

Let us find how many such number are there?

From the above sequence, we know that

$t_n = 995$, $a = 100$

$t_1 = 105$, $t_2 = 110$

Thus, $d = t_2 – t_1 = 110 – 105 = 5$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

we can find value of "n"

Thus, on substituting all the value in formula we get,

$995 = 100 + (n – 1)\times 5$

$\Rightarrow 995 – 100 = (n – 1)\times 5$

$\Rightarrow 895 = (n – 1) \times 5$

$$\Rightarrow n - 1 = \frac{895}{5} = 179$$

$\Rightarrow n = 179 + 1 = 180$

Q. 7. The 11^{th} term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41th term of that A.P.

Answer : Given: $t_{11} = 16$ and $t_{21} = 29$

To find: t_{41}

Using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

we will find value of “a” and “d”

Let, t_{11} = a + (11 – 1) d

⇒ 16 = a + 10 d …..(1)

t_{21} = a + (21 – 1) d

⇒ 29 = a + 20 d …..(2)

Subtracting eq. (1) from eq. (2), we get,

⇒ 29 – 16 = (a – a) + (20 d – 10 d)

⇒ 13 = 10 d

$$\Rightarrow d = \frac{13}{10} = 1.3$$

Substitute value of “d” in eq. (1) to get value of “a”

$$\Rightarrow 16 = a + 10 \times \frac{13}{10}$$

⇒ 16 = a + 13

⇒ a = 16 – 13 = 3

Now, we will find the value of t_{41} using n^{th} term of an A.P. formula

$$\Rightarrow t_{41} = 3 + (41 - 1) \times \frac{13}{10}$$

$$\Rightarrow t_{41} = 3 + 40 \times \frac{13}{10}$$

⇒ t_{41} = 3 + 4 × 13 = 3 + 52 = 55

Thus, t_{41} = 55

Q. 8. 11, 8, 5, 2, . . . In this A.P. which term is number – 151?

Answer : By, given A.P. 11, 8, 5, 2, . . .

we know that

$a = 11$, $t_1 = 8$, $t_2 = 5$

Thus, $d = t_2 – t_1 = 5 – 8 = – 3$

Given: $t_n = – 151$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

we can find value of “n”

Thus, on substituting all the value in formula we get,

$– 151 = 11 + (n – 1)\times (– 3)$

$\Rightarrow – 151 – 11 = (n – 1)\times (– 3)$

$\Rightarrow – 162 = (n – 1) \times (– 3)$

$$\Rightarrow n - 1 = \frac{-162}{-3} = 54$$

$\Rightarrow n = 54 + 1 = 55$

Q. 9. In the natural numbers from 10 to 250, how many are divisible by 4?

Answer : List of number divisible by 4 in between 10 to 250 are

12, 16,20,24,……….. 248

Let us find how many such number are there?

From the above sequence, we know that

$t_n = 248$, $a = 12$

$t_1 = 16$, $t_2 = 20$

Thus, $d = t_2 – t_1 = 20 – 16 = 4$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

we can find value of “n”

Thus, on substituting all the value in formula we get,

248 = 12 + (n – 1)× 4

⇒ 248 – 12 = (n – 1)× 4

⇒ 236 = (n – 1) × 4

⇒ $n - 1 = \frac{236}{4} = 59$

⇒ n = 59 + 1 = 60

Q. 10. In an A.P. 17^{th} term is 7 more than its 10^{th} term. Find the common difference.

Answer :

Given: $t_{17} = 7 + t_{10}$ ……(1)

In t_{17}, n = 17

In t_{10}, n = 10

By using n^{th} term of an A.P. formula,

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ term

Thus, on using formula in eq. (1) we get,

⇒ a + (17 – 1)d = 7 + (a + (10 – 1)d)

⇒ a + 16 d = 7 + (a + 9 d)

⇒ a + 16 d – a – 9 d = 7

⇒ 7 d = 7

$$\Rightarrow d = \frac{7}{7} = 1$$

Thus, common difference “d” = 1

Practice Set 3.3

Q. 1. First term and common difference of an A.P. are 6 and 3 respectively ; find S_{27}.

a = 6, d = 3, S_{27} = ?

$$S_n = \frac{n}{2}[\square + (n-1)d]$$

$$S_{27} = \frac{27}{2}[12 + (27-1)\square]$$

$$= \frac{27}{2} \times \square$$

$$= 27 \times 45 = \boxed{}$$

Answer :

Given: First term a = 6

Common Difference d = 3

To find: S_{27} where n = 27

By using sum of n^{th} term of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, Substituting given value in formula we can find the value of S_{27}

$$\Rightarrow S_{27} = \frac{27}{2}[2 \times 6 + (27-1) \times 3]$$

$$\Rightarrow S_{27} = \frac{27}{2}[12 + 26 \times 3]$$

$$\Rightarrow S_{27} = \frac{27}{2}[12 + 78]$$

$$\Rightarrow S_{27} = \frac{27}{2} \times 90 = 27 \times 45 = 1215$$

Thus, $S_{27} = 1215$

Q. 2. Find the sum of first 123 even natural numbers.

Answer : List of first 123 even natural number is

2,4,6,…….

Where first term a = 2

Second term $t_1 = 4$

Third term $t_2 = 6$

Thus, common difference $d = t_2 - t_1 = 6 - 4 = 2$

n = 123

By using sum of n^{th} term of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, Substituting given value in formula we can find the value of S_n

$$\Rightarrow S_n = \frac{123}{2}[2 \times 2 + (123 - 1) \times 2]$$

$$\Rightarrow S_n = \frac{123}{2}[4 + 122 \times 2]$$

$$\Rightarrow S_n = \frac{123}{2}[4 + 244]$$

$$\Rightarrow S_n = \frac{123}{2} \times 248 = 123 \times 122 = 15252$$

Thus, S_n = 15252

Q. 3. Find the sum of all even numbers from 1 to 350.

Answer : List of even natural number between 1 to 350 is

2,4,6,…….348

Where first term a = 2

Second term t_1 = 4

Third term t_2 = 6

Thus, common difference d = $t_2 - t_1$ = 6 – 4 = 2

t_n = 348 (As we have to find the sum of even numbers between 1 and 350 therefore excluding 350)

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

we can find value of "n" by substituting all the value in formula we get,

$\Rightarrow 348 = 2 + (n - 1) \times 2$

$\Rightarrow 348 - 2 = 2(n - 1)$

$\Rightarrow 346 = 2(n - 1)$

$$\Rightarrow n - 1 = \frac{346}{2} = 173$$

$\Rightarrow n = 173 + 1 = 174$

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, Substituting given value in formula we can find the value of S_n

$$\Rightarrow S_{174} = \frac{174}{2}[2 \times 2 + (174 - 1) \times 2]$$

$$\Rightarrow S_{174} = \frac{174}{2}[4 + 173 \times 2]$$

$$\Rightarrow S_{174} = \frac{174}{2}[4 + 346]$$

$$\Rightarrow S_{174} = \frac{174}{2} \times 350 = 174 \times 175 = 30{,}450$$

Thus, S_{174} = 30,450

Q. 4. In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms.

Answer : Given: $t_{19} = 52$ and $t_{38} = 128$

To find: value of “a” and “d”

Using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

we will find value of “a” and “d”

Let, $t_{19} = a + (19 – 1) d$

$\Rightarrow 52 = a + 18 d$ …..(1)

$t_{38} = a + (38 – 1) d$

$\Rightarrow 128 = a + 37 d$ …..(2)

Subtracting eq. (1) from eq. (2), we get,

$\Rightarrow 128 – 52 = (a – a) + (37 d – 18 d)$

$\Rightarrow 76 = 19 d$

$$\Rightarrow d = \frac{76}{19} = 4$$

Substitute value of “d” in eq. (1) to get value of “a”

$\Rightarrow 52 = a + 18 \times 4$

$\Rightarrow 52 = a + 72$

$\Rightarrow a = 52 – 72 = – 20$

Now, to find value of S_{56} we will using formula of sum of n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, Substituting given value in formula we can find the value of S_n

$$\Rightarrow S_{56} = \frac{56}{2}[2 \times (-20) + (56-1) \times 4]$$

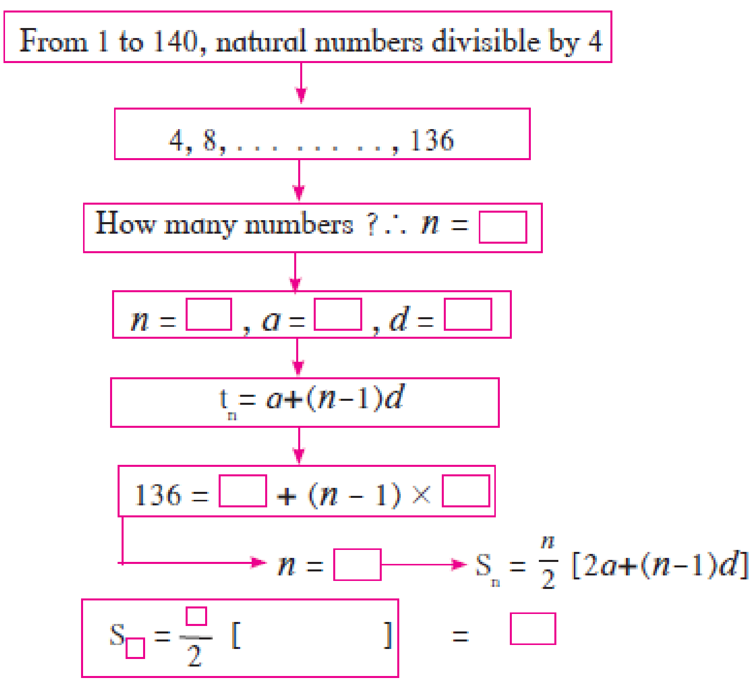
$\Rightarrow S_{56} = 28 \times [-40 + 55 \times 4]$

$\Rightarrow S_{56} = 28 \times [-40 + 220]$

$\Rightarrow S_{56} = 28 \times 180 = 5040$

Thus, $S_{56} = 5040$

Q. 5. Complete the following activity to find the sum of natural numbers from 1 to 140 which are divisible by 4.



Sum of numbers from 1 to 140, which are divisible by 4 = ☐

Answer : List of natural number divisible by 4 between 1 to 140 is

4,8,12,…….136

Where first term a = 4

Second term t_1 = 8

Third term t_2 = 12

Thus, common difference d = $t_2 – t_1$ = 12 – 8 = 4

t_n = 136

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

we can find value of “n” by substituting all the value in formula we get,

⇒ 136 = 4 + (n – 1) × 4

⇒ 136 – 4 = 4(n – 1)

⇒ 132 = 4(n – 1)

$$\Rightarrow n - 1 = \frac{132}{4} = 33$$

⇒ n = 33 + 1 = 34

Now, By using sum of n^{th} term of an A.P. we will find it’s sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, Substituting given value in formula we can find the value of S_{34}

$$\Rightarrow S_{34} = \frac{34}{2}[2 \times 4 + (34-1) \times 4]$$

$\Rightarrow S_{34} = 17 \times [8 + 33 \times 4]$

$\Rightarrow S_{34} = 17 \times [8 + 132]$

$\Rightarrow S_{34} = 17 \times 140 = 2380$

Thus, $S_{34} = 2380$

Q. 6. Sum of first 55 terms in an A.P. is 3300, find its 28th term.

Answer :

Given: $S_{55} = 3300$ where n = 55

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{55} = \frac{55}{2}[2a + (55-1)d]$$

$$\Rightarrow 3300 = \frac{55}{2}[2a + 54d]$$

$$\Rightarrow 3300 = \frac{55}{2} \times 2 \times [a + 27d]$$

⇒ 3300 = 55 × [a + 27d]

$$\Rightarrow \frac{3300}{55} = a + 27d$$

⇒ a + 27d = 60 ……(1)

We need to find value of 28th term i.e t_{28}

Now, By using nth term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = nth terms

we can find value of t_{28} by substituting all the value in formula we get,

⇒ t_{28} = a + (28 – 1) d

⇒ t_{28} = a + 27 d

From eq. (1) we get,

⇒ t_{28} = a + 27 d = 60

⇒ t_{28} = 60

Q. 7. In an A.P. sum of three consecutive terms is 27 and their product is 504 find the terms? (Assume that three consecutive terms in A.P. are a – d, a, a + d.)

Answer :

Let the first term be a – d

the second term be a

the third term be a + d

Given: sum of consecutive three term is 27

⇒ $(a - d) + a + (a + d) = 27$

⇒ $3a = 27$

⇒ $a = \frac{27}{3} = 9$

Also, Given product of three consecutive term is 504

⇒ $(a - d) \times a \times (a + d) = 504$

⇒ $(9 - d) \times 9 \times (9 + d) = 504$ (since, a = 9)

⇒ $(9 - d) \times (9 + d) = \frac{504}{9} = 56$

⇒ $9^2 - d^2 = 56$ (since, $(a - b)(a + b) = a^2 - b^2$)

⇒ $81 - d^2 = 56$

⇒ $d^2 = 81 - 56 = 25$

⇒ $d = \sqrt{25} = \pm 5$

Case 1:

Thus, if a = 9 and d = 5

Then the three terms are,

First term a – d = 9 – 5 = 4

Second term a = 9

Third term a + d = 9 + 5 = 14

Thus, the A.P. is 4, 9, 14

Case 2:

Thus, if a = 9 and d = – 5

Then the three terms are,

First term a – d = 9 – (– 5) = 9 + 5 = 14

Second term a = 9

Third term a + d = 9 + (– 5) = 9 – 5 = 4

Thus, the A.P. is 14, 9, 4

Q. 8. Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14.

(Assume the four consecutive terms in A.P. are a – d, a, a + d, a + 2d.)

Answer : Let the first term be a – d

the second term be a

the third term be a + d

the fourth term be a + 2 d

Given: sum of consecutive four term is 12

⇒ (a – d) + a + (a + d) + (a + 2d) = 12

⇒ 4 a + 2d = 12

⇒ 2(2 a + d) = 12

$\Rightarrow 2a + d = \frac{12}{2} = 6$

⇒ 2a + d = 6 …..(1)

Also, sum of third and fourth term is 14

⇒ (a + d) + (a + 2d) = 14

⇒ 2a + 3d = 14 ……(2)

Subtracting eq. (1) from eq. (2) we get,

⇒(2a + 3d) – (2a + d) = 14 – 6

$\Rightarrow 2a + 3d - 2a - d = 8$

$\Rightarrow 2d = 8$

$$\Rightarrow d = \frac{8}{2} = 4$$

$\Rightarrow d = 4$

Substituting value of “d” in eq. (1) we get,

$\Rightarrow 2a + 4 = 6$

$\Rightarrow 2a = 6 - 4 = 2$

$$\Rightarrow a = \frac{2}{2} = 1$$

$\Rightarrow a = 1$

Thus, a = 1 and d = 4

Hence, first term a – d = 1 – 4 = – 3

the second term a = 1

the third term a + d = 1 + 4 = 5

the fourth term a + 2 d = 1 + 2×4 = 1 + 8 = 9

Thus, the A.P. is – 3, 1, 5, 9

Q. 9. If the 9th term of an A.P. is zero then show that the 29th term is twice the 19th term.

Answer : Now, By using nth term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = nth terms

Given: $t_9 = 0$

$\Rightarrow t_9 = a + (9 - 1)d$

$\Rightarrow 0 = a + 8d$

$\Rightarrow a = -8d$

To Show: $t_{29} = 2\times t_{19}$

Now,

$\Rightarrow t_{29} = a + (29 - 1)d$

$\Rightarrow t_{29} = a + 28d$

$\Rightarrow t_{29} = -8d + 28d = 20d$ (since, $a = -8d$)

$\Rightarrow t_{29} = 20d$

$\Rightarrow t_{29} = 2 \times 10d$ ….(1)

Also,

$\Rightarrow t_{19} = a + (19 - 1)d$

$\Rightarrow t_{19} = a + 18d$

$\Rightarrow t_{19} = -8d + 18d = 10d$ (since, $a = -8d$)

$\Rightarrow t_{19} = 10d$ …..(2)

From eq. (1) and eq. (2) we get,

$t_{29} = 2\times t_{19}$

Practice Set 3.4

Q. 1. On 1st Jan 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st Dec 2016 what would be her total saving?

Answer : By given information we can form an A.P.

10, 11, 12, 13, ……

Hence, the first term a = 10

Second term t_1 = 11

Third term t_2 = 12

Thus, common difference d = $t_2 - t_1$ = 12 – 11 = 1

Here, number of terms from 1st Jan 2016 to 31st Dec 2016 is,

n = 366

We need to find S_{366}

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{366} = \frac{366}{2}[2 \times 10 + (366-1) \times 1]$$

$\Rightarrow S_{366}$ = 183 [20 + 365]

$\Rightarrow S_{366}$ = 183 × 385

$\Rightarrow S_{366}$ = Rs 70,455

Q. 2. A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.

Answer : Given: A man borrows = Rs. 8000

Repay with total interest = Rs 1360

In 12 months, thus n = 12

Thus, S_{12} = 8000 + 1360 = 9360

Each installment being less than preceding one

Thus, d = – 40

We need to find "a"

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{12} = \frac{12}{2}[2a + (12-1)\times(-40)]$$

⇒ 9360 = 6 [2a – 11 × 40]

$$\Rightarrow \frac{9360}{6} = 2a - 440$$

⇒ 1560 = 2a – 440

⇒ 1560 + 440 = 2a

⇒ 2a = 2000

$$\Rightarrow a = \frac{2000}{2} = 1000$$

Thus, first installment a = Rs. 1000

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$

Where, n = no. of terms

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

Let a = first term, t_n = last term

$$\Rightarrow S_{12} = \frac{12}{2}[a + t_n]$$

⇒ 9360 = 6 [1000 + t_n]

$$\Rightarrow 1000 + t_n = \frac{9360}{6} = 1560$$

⇒ t_n = 1560 – 1000 = 560

Thus, last installment t_n = 560

Q. 3. Sachin invested ina national saving certificate scheme. In the first year he invested ₹ 5000, in the second year ₹ 7000, in the third year ₹ 9000 and so on. Find the total amount that he invested in 12 years.

Answer : By given information we can form an A.P.

5000, 7000, 9000, ……

Hence, the first term a = 5000

Second term t_1 = 7000

Third term t_2 = 9000

Thus, common difference d = t_2 – t_1 = 9000 – 7000 = 2000

Here, number of terms n = 12

We need to find S_{12}

Now, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{12} = \frac{12}{2}[2 \times 5000 + (12-1) \times 2000]$$

$\Rightarrow S_{12} = 6 \times [10,000 + 11 \times 2000]$

$\Rightarrow S_{12} = 6 \times [10,000 + 22,000]$

$\Rightarrow S_{12} = 6 \times 32,000$

$\Rightarrow S_{12}$ = Rs. 192000

Q. 4. There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the 15th row and also find how many total seats are there in the auditorium?

Answer : Given: first term a = 20

Second term t_1 = 22

Third term t_2 = 24

Common difference $d = t_2 - t_1 = 24 - 22 = 2$

We need to find t_{15} thus n = 15

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

On substituting all value in n^{th} term of an A.P.

⇒ $t_{15} = 20 + (15 – 1) \times 2$

⇒ $t_{15} = 20 + 14 \times 2$

⇒ $t_{15} = 20 + 28 = 48$

We have been given that, there are 27 rows in an auditorium

Thus, we need to find total seats in auditorium i.e. S_{27}

Now, By using sum of n^{th} term of an A.P. we will find it’s sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{27} = \frac{27}{2}[2 \times 20 + (27-1) \times 2]$$

$$\Rightarrow S_{27} = \frac{27}{2} \times 2 \times [20 + 26]$$

⇒$S_{27} = 27 \times 46$

⇒$S_{27} = 1242$

Q. 5. Kargil’s temperature was recorded in a week from Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5° C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was – 30° celsius then find the temperature on the other five days.

Answer :

Let Monday be the first term i.e. $a = t_1$

Let Tuesday be the second term i.e t_2

Let Wednesday be the third term i.e t_3

Let Thursday be the fourth term i.e t_4

Let Friday be the fifth term i.e t_5

Let Saturday be the sixth term i.e t_6

Given: $t_1 + t_6 = 5 + (t_2 + t_6)$

$\Rightarrow a = 5 + (t_2 + t_6) – t_6$

$\Rightarrow a = 5 + t_2$ …..(1)

We know that,

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Thus, $t_2 = a + (2 – 1)\ d$

$\Rightarrow t_2 = a + d$

Now substitute value of t_2 in (1) we get,

$\Rightarrow a = 5 + (a + d)$

$\Rightarrow d = a – 5 – a = – 5$

Given: $t_3 = – 30°$

Thus, $t_3 = a + (3 - 1) \times (-5)$

$\Rightarrow -30 = a + 2 \times (-5)$

$\Rightarrow -30 = a - 10$

$\Rightarrow a = -30 + 10 = -20°$

Thus, Monday, $a = t_1 = -20°$

Using formula $t_{n+1} = t_n + d$

We can find the value of the other terms

Tuesday, $t_2 = t_1 + d = -20 - 5 = -25°$

Wednesday, $t_3 = t_2 + d = -25 - 5 = -30°$

Thursday, $t_4 = t_3 + d = -30 - 5 = -35°$

Friday, $t_5 = t_4 + d = -35 - 5 = 40°$

Saturday, $t_6 = t_5 + d = -40 - 5 = -45°$

Thus, we obtain an A.P.

$-20°, -25°, -30°, -35°, -40°, -45°$

Q. 6. On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Find the total number of trees in the 25 rows.

Answer : First term a = 1

Second term $t_1 = 2$

Third term $t_3 = 3$

Common difference $d = t_3 - t_2 = 3 - 2 = 1$

We need to find total number of trees when n = 25

Thus, By using sum of n^{th} term of an A.P. we will find it’s sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{25}

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{25} = \frac{25}{2}[2 \times 1 + (25-1) \times 1]$$

$$\Rightarrow S_{25} = \frac{25}{2}[2 + 24]$$

$$\Rightarrow S_{25} = \frac{25}{2} \times 2 \times [1 + 12]$$

$\Rightarrow S_{25} = 25 \times 13 = 325$

Problem Set 3

Q. 1 A. Choose the correct alternative answer for each of the following sub questions.

The sequence – 10, – 6, – 2, 2, . . .

A. is an A.P., Reason d = – 16
B. is an A.P., Reason d = 4
C. is an A.P., Reason d = – 4
D. is not an A.P.

Answer :

First term a = – 10

Second term t_1 = – 6

Third term t_2 = – 2

Fourth term $t_3 = 2$

Common difference $d = t_1 – a = – 6 – (– 10) = – 6 + 10 = 4$

Common difference $d = t_2 – t_1 = – 2 – (– 6) = – 2 + 6 = 4$

Common difference $d = t_3 – t_2 = 2 – (– 2) = 2 + 2 = 4$

Since, the common difference is same

∴ The given sequence is A.P. with common difference d = 4

Hence, correct answer is (B)

Q. 1 B. Choose the correct alternative answer for each of the following sub questions.

First four terms of an A.P. are ....., whose first term is – 2 and commondifference is – 2.

A. – 2, 0, 2, 4
B. – 2, 4, – 8, 16
C. – 2, – 4, – 6, – 8
D. – 2, – 4, – 8, – 16

Answer : Given first term $t_1 = – 2$

Common difference d = – 2

By using formula $t_{n+1} = t_n + d$

$t_2 = t_1 + d = – 2 + (– 2) = – 2 – 2 = – 4$

$t_3 = t_2 + d = – 4 + (– 2) = – 4 – 2 = – 6$

$t_4 = t_3 + d = – 6 + (– 2) = – 6 – 2 = – 8$

Hence, the A.P. is – 2, – 4, – 6, – 8

∴ correct answer is (C)

Q. 1 C. Choose the correct alternative answer for each of the following sub questions.

What is the sum of the first 30 natural numbers ?

A. 464
B. 465
C. 462
D. 461

Answer : List of first 30 natural number is

1, 2, 3,……..,30

First term a = 1

Second term t_1 = 2

Third term t_2 = 3

Common difference d = $t_3 - t_2$ = 3 – 2 = 1

number of terms n = 30

Thus, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{30}

$$\Rightarrow S_{30} = \frac{30}{2}[2 \times 1 + (30-1) \times 1]$$

$\Rightarrow S_{30}$ = 15 [2 + 29]

$\Rightarrow S_{30} = 15 \times 31$

$\Rightarrow S_{30} = 465$

Hence, Correct answer is (B)

Q. 1 D. Choose the correct alternative answer for each of the following sub questions.

For an given A.P. $t_7 = 4$, $d = -4$ then a = . . .

A. 6
B. 7
C. 20
D. 28

Answer :

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

$\Rightarrow t_7 = a + (7 - 1) \times (-4)$

$\Rightarrow 4 = a + 6 \times (-4)$

$\Rightarrow 4 = a - 24$

$\Rightarrow a = 24 + 4 = 28$

Thus, the correct answer is (D)

Q. 1 E. Choose the correct alternative answer for each of the following sub questions.

For an given A.P. a = 3.5, d = 0, n = 101, then t_n = . . .

A. 0
B. 3.5
C. 103.5
D. 104.5

Answer :

Given: $a = 3.5$, $d = 0$, $n = 101$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Substituting all given value in the formulae we get,

$\Rightarrow t_n = 3.5 + (101 – 1) \times 0$

$\Rightarrow t_n = 3.5$

Thus, correct answer is (B)

Q. 1 F. Choose the correct alternative answer for each of the following sub questions.

In an A.P. first two terms are – 3, 4 then 21^{st} term is . . .

A. – 143
B. 143
C. 137
D. 17

Answer :

Given: first term $a = – 3$

Second term $t_1 = 4$

Common difference $d = t_1 – a = 4 – (– 3) = 4 + 3 = 7$

We need to find t_{21} where n = 21

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Substituting all given value in the formulae we get,

$\Rightarrow t_{21} = – 3 + (21 – 1) \times 7$

$\Rightarrow t_{21} = – 3 + 20 \times 7$

$\Rightarrow t_{21} = – 3 + 140 = 137$

Hence, correct answer is (C)

Q. 1 G. Choose the correct alternative answer for each of the following sub questions.

If for any A.P. d = 5 then $t_{18} – t_{13}$ = ...

A. 5
B. 20
C. 25
D. 30

Answer :

Given d = 5

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

Thus, $t_{18} – t_{13}$ = [a + (18 – 1) × 5] – [a + (13 – 1) × 5]

⇒ $t_{18} – t_{13}$ = [17 × 5] – [12 × 5]

⇒ $t_{18} – t_{13}$ = 85 – 60 = 25

Thus, correct answer is (C)

Q. 1 H. Choose the correct alternative answer for each of the following sub questions.

Sum of first five multiples of 3 is. . .

A. 45
B. 55
C. 15
D. 75

Answer :

First five multiples of 3 are

3, 6, 9, 12, 15

First term a = 3

Second term t_1 = 6

Third term t_2 = 9

Common difference d = $t_2 – t_1$ = 9 – 6 = 3

Thus, By using sum of n^{th} term of an A.P. we will find it’s sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_5

$$\Rightarrow S_5 = \frac{5}{2}[2 \times 3 + (5-1) \times 3]$$

$$\Rightarrow S_5 = \frac{5}{2}[6 + 4 \times 3]$$

$$\Rightarrow S_5 = \frac{5}{2}[6 + 12]$$

$$\Rightarrow S_5 = \frac{5}{2} \times 18 = 5 \times 9 = 45$$

Thus, correct answer is (A)

Q. 1 I. Choose the correct alternative answer for each of the following sub questions.

15, 10, 5, . . . In this A.P. sum of first 10 terms is . . .

A. – 75
B. – 125
C. 75
D. 125

Answer :

First term a = 15

Second term t_1 = 10

Third term t_2 = 5

Common difference d = $t_2 - t_1$ = 5 – 10 = – 5

No. of terms n = 10

Thus, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{10}

$$\Rightarrow S_{10} = \frac{10}{2}\,[\,2 \times 15 + (10-1) \times (-5)]$$

$\Rightarrow S_{10} = 5\,[\,30 + 9 \times (-5)]$

$\Rightarrow S_{10} = 5\,[\,30 - 45]$

$\Rightarrow S_{10} = 5 \times (-15) = -75$

Hence, correct answer is (A)

Q. 1 J. Choose the correct alternative answer for each of the following sub questions.

In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is = 399 then n = . . .
A. 42
B. 38
C. 21
D. 19

Answer :

Given, first term = 1

Last term = 20

Sum of n terms, S_n = 399

We need to find no. of terms n

Using Sum of n terms of an A.P. formula

$$S_n = \frac{n}{2}\,[\,\text{first term} + \text{last term}]$$

where n = no. of terms

S_n = sum of n terms

Now, on substituting given value in formula we get,

$\Rightarrow 399 = \frac{n}{2}[1 + 20]$

$\Rightarrow 399 = \frac{n}{2} \times 21$

$\Rightarrow n = \frac{399 \times 2}{21} = 19 \times 2 = 38$

∴ correct answer is (B)

Q. 2. Find the fourth term from the end in an A.P. – 11, – 8, – 5, . . ., 49.

Answer : First term from end a = 49

$t_n = -11$

$t_{n-1} = -8$

Common difference $d = t_n - t_{n-1} = -11 - (-8) = -11 + 8 = -3$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

no. of terms n = 4

$\Rightarrow t_4 = 49 + (4 - 1) \times (-3)$

$\Rightarrow t_4 = 49 + 3 \times (-3)$

$\Rightarrow t_4 = 49 - 9 = 40$

Q. 3. In an A.P. the 10th term is 46, sum of the 5th and 7th term is 52. Find the A.P.

Answer : Given: $t_{10} = 46$

$t_5 + t_7 = 52$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Hence, by given condition we get,

$\Rightarrow t_{10} = 46$

$\Rightarrow a + (10 - 1)d = 46$

$\Rightarrow a + 9d = 46$ ……(1)

$\Rightarrow t_5 + t_7 = 52$

$\Rightarrow [a + (5 - 1)d] + [a + (7 - 1)d] = 52$

$\Rightarrow [a + 4d] + [a + 6d] = 52$

$\Rightarrow 2a + 10d = 52$ ……(2)

Multiply eq. (2) by 2 we get,

$\Rightarrow 2a + 18d = 92$ ……(3)

Subtract eq. (2) by eq. (3)

$\Rightarrow [2a + 18d] – [2a + 10d] = 92 – 52$

$\Rightarrow 8d = 40$

$$\Rightarrow d = \frac{40}{8} = 5$$

Substitute “d” in (1)

$\Rightarrow a + 9 \times 5 = 46$

$\Rightarrow a + 45 = 46$

$\Rightarrow a = t_1 = 46 – 45 = 1$

we know that, $t_{n+1} = t_n + d$

$\Rightarrow t_2 = t_1 + d = 1 + 5 = 6$

$\Rightarrow t_3 = t_2 + d = 6 + 5 = 11$

Hence, an A.P. is 1, 6, 11, . . .

Q. 4. The A.P. in which 4th term is – 15 and 9th term is – 30. Find the sum of the first 10 numbers.

Answer : $t_4 = -15$ and $t_9 = -30$

Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Hence, by given condition we get,

$t_4 = -15$

$\Rightarrow a + (4 - 1)d = -15$

$\Rightarrow a + 3d = -15$ …..(1)

$t_9 = -30$

$\Rightarrow a + (9 - 1)d = -30$

$\Rightarrow a + 8d = -30$ …..(2)

Subtracting eq. (1) from eq. (2)

$\Rightarrow [a + 8d] - [a + 3d] = -30 - (-15)$

$\Rightarrow 5d = -30 + 15 = -15$

$$\Rightarrow d = -\frac{15}{5} = -3$$

Substituting, "d" in eq. (1)

$\Rightarrow a + 3 \times (-3) = -15$

$\Rightarrow a + -9 = -15$

$\Rightarrow a = -15 + 9 = -6$

Thus, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{10}

$$\Rightarrow S_{10} = \frac{10}{2}[2 \times (-6) + (10-1) \times (-3)]$$

$\Rightarrow S_{10} = 5 [- 12 + 9 \times (- 3)]$

$\Rightarrow S_{10} = 5 [- 12 - 27]$

$\Rightarrow S_{10} = 5 \times (- 39) = - 195$

Q. 5. Two A.P.'s are given 9, 7, 5, . . . and 24, 21, 18, ... If nth term of both the progressions are equal then find the value of n and nth term.

Answer : Given A.P. is 9, 7, 5….

Whose first tern a = 9

Second term $t_1 = 7$

Third term $t_3 = 5$

Common difference $d = t_3 - t_2 = 5 - 7 = - 2$

Another A.P. is 24, 21, 18, . . .

Whose first tern a = 24

Second term $t_1 = 21$

Third term $t_3 = 18$

Common difference $d = t_3 - t_2 = 18 - 21 = - 3$

We have been given, n^{th} term of both the A.P. is same

thus, by using n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

Hence, by given condition we get,

⇒ 9 + (n – 1) × (– 2) = 24 + (n – 1) × (– 3)

⇒ 9 – 2n + 2 = 24 – 3n + 3

⇒ 11 – 2n = 27 – 3n

⇒ 3n – 2n = 27 – 11

⇒ n = 16

Thus, value of n^{th} term where a = 9, d = – 2, n = 16 is

⇒ t_n = 9 + (16 – 1) × (– 2)

⇒ t_n = 9 – 15 × 2

⇒ t_n = 9 – 30 = – 21

Q. 6. If sum of 3rd and 8th terms of an A.P. is 7 and sum of 7th and 14th terms is – 3 then find the 10th term.

Answer : Now, By using n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

Hence, by given condition we get,

$t_3 + t_8 = 7$

$\Rightarrow [a + (3 - 1)d] + [a + (8 - 1)d] = 7$

$\Rightarrow [a + 2d] + [a + 7d] = 7$

$\Rightarrow 2a + 9d = 7$ .....(1)

$t_7 + t_{14} = -3$

$\Rightarrow [a + (7 - 1)d] + [a + (14 - 1)d] = -3$

$\Rightarrow [a + 6d] + [a + 13d] = -3$

$\Rightarrow 2a + 19d = -3$ .....(2)

Subtracting eq. (1) from eq. (2)

$\Rightarrow [2a + 19d] - [2a + 9d] = -3 - 7$

$\Rightarrow 10d = -10$

$$\Rightarrow d = -\frac{10}{10} = -1$$

Substituting, "d" in eq. (1)

$\Rightarrow 2a + 9 \times (-1) = 7$

$\Rightarrow 2a - 9 = 7$

$\Rightarrow 2a = 7 + 9 = 16$

$$\Rightarrow a = \frac{16}{2} = 8$$

Now, we can find value of t_{10}

Thus, $t_{10} = 8 + (10 - 1) \times (-1)$

Q. 7 In an A.P. the first term is – 5 and last term is 45. If sum of all numbers in the A.P. is 120, then how many terms are there? What is the common difference?

Answer :

Given, first term a = – 5

Last term t_n = 45

Sum of n terms S_n = 120

To find no of terms "n"

Using Sum of n terms of an A.P. formula

$$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$

where n = no. of terms

S_n = sum of n terms

Now, on substituting given value in formula we get,

$$\Rightarrow 120 = \frac{n}{2}[-5 + 45]$$

$$\Rightarrow 120 = \frac{n}{2} \times 40$$

⇒ 120 = 20 n

$$\Rightarrow n = \frac{120}{20} = 6$$

To find the common difference 'd'

We use n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

Thus, on substituting all values we get,

⇒ $t_6 = -5 + (6 - 1)d$

⇒ $45 = -5 + 5d$

⇒ $5d = 45 + 5 = 50$

$$\Rightarrow d = \frac{50}{5} = 10$$

Thus, common difference is 10

Q. 8. Sum of 1 to n natural numbers is 36, then find the value of n.

Answer : List of n natural number is

1, 2, 3, ……n

First term a = 1

Second term $t_1 = 2$

Third term $t_3 = 3$

Thus, common difference $d = t_3 - t_2 = 3 - 2 = 1$

Given $S_n = 36$

Thus, By using sum of n^{th} term of an A.P. we will find it's sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find no. of terms n

$$\Rightarrow 36 = \frac{n}{2}[2 \times 1 + (n-1) \times 1]$$

$$\Rightarrow 36 = \frac{n}{2}[2 + n - 1]$$

$$\Rightarrow 36 = \frac{n}{2}[1 + n]$$

$\Rightarrow n(1 + n) = 36 \times 2 = 72$

$\Rightarrow n^2 + n - 72 = 0$

$\Rightarrow n^2 + 9n - 8n - 72 = 0$

$\Rightarrow n(n + 9) - 8(n + 9) = 0$

$\Rightarrow (n - 8)(n + 9) = 0$

$\Rightarrow n - 8 = 0$ or $n + 9 = 0$

$\Rightarrow n = 8$ or $n = -9$

Since, number of terms n can't be negative

$\therefore n = 8$

Q. 9. Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623.

Answer : Let 3 parts of 207 be a – d, a, a + d such that,

$\Rightarrow (a - d) + a + (a + d) = 207$

$\Rightarrow 3a = 207$

$$\Rightarrow a = \frac{207}{3} = 69$$

Since, product of two smaller terms is 4623

⇒ (a – d) × a = 4623

⇒ (69 – d) × 69 = 4623

$$\Rightarrow 69 - d = \frac{4623}{69} = 67$$

⇒ d = 69 – 67 = 2

Thus, a – d = 69 – 2 = 67

a = 69

a + d = 69 + 2 = 71

Thus, the A.P so formed is 67, 69, 71

Q. 10. There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three terms is 429. Write the A.P.

Answer : Let first term = a

Common difference = d

Since, A.P. consist of 37 terms, therefor the middle most term is

$$\frac{37 + 1}{2} = \frac{38}{2} = 19\text{th term}$$

Thus, three middle most term are t_{18} = 18^{th} term, t_{19} = 19^{th} term,

t_{20} = 20^{th} term

We use n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

Thus, on substituting all values we get,

Given, $t_{18} + t_{19} + t_{20} = 225$

$\Rightarrow [a + (18 - 1)d] + [a + (19 - 1)d] + [a + (20 - 1)d] = 225$

$\Rightarrow [a + 17d] + [a + 18d] + [a + 19d] = 225$

$\Rightarrow 3a + 54d = 225$

Dividing by 3

$\Rightarrow a + 18d = 75$ …….(1)

Given, sum of last three term is 429

$\Rightarrow t_{35} + t_{36} + t_{37} = 429$

$\Rightarrow [a + (35 - 1)d] + [a + (36 - 1)d] + [a + (37 - 1)d] = 429$

$\Rightarrow [a + 34d] + [a + 35d] + [a + 36d] = 429$

$\Rightarrow 3a + 105d = 429$

Dividing by 3

$a + 35d = 143$ …….(2)

Subtracting eq. (1) from eq. (2) we get,

$\Rightarrow [a + 35d] - [a + 18d] = 143 - 75$

$\Rightarrow 17d = 68$

$$\Rightarrow d = \frac{68}{17} = 4$$

Substituting value of ‘d’ in eq. (1) we get,

$\Rightarrow a + 18 \times 4 = 75$

$\Rightarrow a + 72 = 75$

$\Rightarrow a = 75 - 72 = 3$

$\Rightarrow a = t_1 = 3$

We know that, $t_{n+1} = t_n + d$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_{37} = 3 + (37 - 1) \times 4$

$t_{37} = 3 + 36 \times 4$

$t_{37} = 3 + 144 = 147$

Thus, the A.P. is 3, 7, 11, ..., 147

Q. 11. If first term of an A.P. is a, second term is b and last term is c, then show that sum of all terms is

$$\frac{(a+c)(b+c-2a)}{2(b-a)}.$$

Answer :

Given first term = a

Second term = b

Last term = c

Common difference d = second term – first term = b – a

We will first find the number of terms

We use n^{th} term of an A.P. formula

$t_n = a + (n – 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Thus, on substituting all values we get,

$\Rightarrow c = a + (n – 1)(b – a)$

$\Rightarrow c = a + (b – a)n + a – b$

$c = 2a – b + (b – a)n$

$\Rightarrow (b – a)n = c + b – 2a$

$$\Rightarrow n = \frac{c + b - 2a}{b - a}$$

Using Sum of n terms of an A.P. formula

$$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$

where n = no. of terms

S_n = sum of n terms

On substituting all the values we get,

$$\Rightarrow S_n = \frac{c + b - 2a}{2(b - a)}[a + c]$$

$$\Rightarrow S_n = \frac{(a + c)(c + b - 2a)}{2(b - a)}$$

Hence, proved

Q. 12. If the sum of first p terms of an A.P. is equal to the sum of first q terms then show that the sum of its first (p + q) terms is zero. (p ≠ q)

Answer : We know that, sum of n^{th} term of an A.P. we will find it's

sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Now, Sum of p terms is

$$S_p = \frac{p}{2}[2a + (p-1)d]$$

And, Sum of q terms is

$$S_q = \frac{q}{2}[2a + (q-1)d]$$

Given: $S_p = S_q$

$$\Rightarrow \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$

Multiply by 2 on both sides, we get,

$\Rightarrow p[2a + (p-1)d] = q[2a + (q-1)d]$

$\Rightarrow 2ap + p(p-1)d = 2aq + q(q-1)d$

$\Rightarrow 2ap - 2aq + p(p-1)d - q(q-1)d = 0$

$\Rightarrow 2a(p-q) + d[p^2 - p - q^2 + q] = 0$

$\Rightarrow 2a(p - q) + d[(p^2 - q^2) - p + q] = 0$

$2a(p - q) + d[(p - q)(p + q) - (p - q)] = 0$

(since, $(a - b)(a + b) = a^2 - b^2$)

$\Rightarrow 2a(p - q) + d(p - q)[p + q - 1] = 0$

$\Rightarrow (p - q)[2a + d(p + q - 1)] = 0$

Since, $p \neq q$

$\therefore p - q \neq 0$

$\Rightarrow 2a + d(p + q - 1) = 0$

Multiply both side by $\frac{p + q}{2}$

$$\Rightarrow \frac{p + q}{2}[2a + d(p + q - 1)] = 0$$

$\Rightarrow S_{p+q} = 0$

Hence proved

Q. 13. If m times the mth term of an A.P. is equal to n times nth term then show that the (m + n)th term of the A.P. is zero.

Answer :

We use n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

Thus m^{th} term $= t_m = a + (m - 1)d$

Given: $m \times t_m = n \times t_n$

$\Rightarrow m \times [\ a + (m - 1)d] = n \times [a + (n - 1)d]$

$\Rightarrow am + m(m - 1)d = an + n(n - 1)d$

$\Rightarrow am - an + m(m - 1)d - n(n - 1)d = 0$

$\Rightarrow a(m - n) + d[m(m - 1) - n(n - 1)] = 0$

$\Rightarrow a(m - n) + d[\ m^2 - m - n^2 + n] = 0$

$\Rightarrow a(m - n) + d[\ (m^2 - n^2) - m + n] = 0$

$\Rightarrow a(m - n) + d[\ (m - n)(m + n) -(m - n)] = 0$

(since, $(a - b)(a + b) = a^2 - b^2$)

$\Rightarrow a(m - n) + d(m - n)[(m + n) -1] = 0$

$\Rightarrow (m - n)\ [a + d(m + n -1)] = 0$

Since, $m \neq n$

$\therefore m - n \neq 0$

$\Rightarrow a + d(m + n -1) = 0$

$\Rightarrow t_{m+n} = 0$

Hence proved

Q. 14. ₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.

Simple interest $= \frac{P \times R \times N}{100}$

Simple interest after 1 year $= \frac{1000 \times 10 \times 1}{100} = \square$

Simple interest after 2 year $= \frac{1000 \times 10 \times 2}{100} = \square$

Simple interest after 3 year $= \frac{\square \times \square \times \square}{100} = 300$

According to this the simple interest for 4, 5, 6 years will be 400, $\square$, $\square$ respectively.

From this $d = \square$ and $a = \square$

Amount of simple interest after 20 years

$t_n + a + (n - 1)\, d$

$t_{20} = \square + (20-1)\square$

$t_{20} = \square$

Amount of simple interest after 20 years is = $\square$

Answer : Given: Principal Amount P = 1000

Rate of interest R = 10%

Also, Simple Interest $= \frac{(P \times R \times N)}{100}$

Simple interest after 1 year $= \frac{1000 \times 10 \times 1}{100} = 100$

Simple interest after 2 year $= \frac{1000 \times 10 \times 2}{100} = 200$

Simple interest after 3 year $= \frac{1000 \times 10 \times 3}{100} = 300$

According to this the simple interest for 4, 5, 6 years will be 400,

500, 600 respectively.

Let first term a = 100

Second term t_1 = 200

Third term t_3 = 300

Common difference d = $t_3 - t_2$ = 300 – 200 = 100

Amount of simple interest after 20 years

We use n^{th} term of an A.P. formula

$t_n = a + (n - 1)d$

where n = no. of terms

a = first term

d = common difference

t_n = n^{th} terms

$\Rightarrow t_{20} = 100 + (20 - 1) \times 100$

$\Rightarrow t_{20} = 100 + 19 \times 100$

$\Rightarrow t_{20} = 100 + 1900 = 2000$