Circle

Practice Set 3.1

Q. 1. In the adjoining figure the radius of a circle with centre C is 6 cm, line AB is a tangent at A. Answer the following questions.

(1) What is the measure of ∠CAB? Why?
(2) What is the distance of point C from line AB? Why?
(3) d(A,B) = 6 cm, find d(B,C).
(4) What is the measure of ∠ABC? Why?

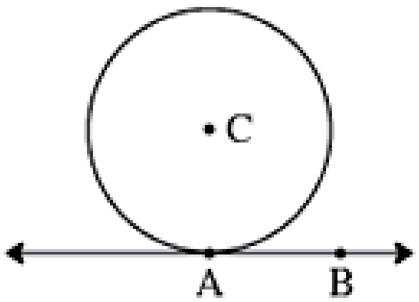


Fig. 3.19

Answer : (1) ere CA is the radius of the circle and A is the point of contact of the tangent AB.

⇒ ∠CAB = 90° Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.

(2) CA is the radius of the circle which is perpendicular to the tangent AB.

So, the perpendicular distance of line AB from C = CA = 6 cm

(3) In triangle ABC right-angled at A,

Given AB = 6 cm and CA = 6 cm

$BC^2 = AB^2 + CA^2$ {Using Pythagoras theorem}

⇒ $BC^2 = 6^2 + 6^2$

⇒ $BC^2 = 36 + 36$

⇒ $BC = \sqrt{72}$

$\Rightarrow BC = 6\sqrt{2}$ cm

(4) In triangle ABC right-angled at A,

AB = CA = 6 cm

$\Rightarrow \angle ABC = \angle ACB$ {Angles opposite to equal sides are equal}

$\Rightarrow \angle ABC + \angle ACB + \angle BAC = 180°$ {Angle sum property of the triangle}

$\Rightarrow 2\angle ABC = 90°$ {$\because \angle BAC = 90°$}

$\Rightarrow \angle ABC = 45°$

Q. 2. In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If (OR) = 10 cm and radius of the circle = 5 cm, then

(1) What is the length of each tangent segment?
(2) What is the measure of ∠MRO?
(3) What is the measure of ∠MRN?

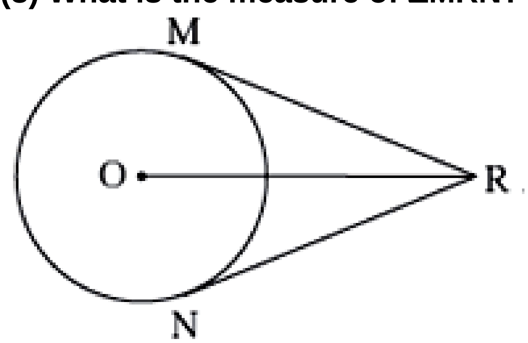


Fig. 3.20

Answer : (1) Here OM is the radius of the circle and M and N are the points of contact of MR and NR respectively.

$\Rightarrow \angle RMO = 90°$ Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.

In triangle ORM right-angled at M,

Given that OR = 10 cm and OM = 5 cm {Radius of the circle}

$OR^2 = OM^2 + RM^2$ {Using Pythagoras theorem}

$\Rightarrow MR^2 = 10^2 - 5^2$

$\Rightarrow MR^2 = 100 - 25$

$\Rightarrow MR = \sqrt{75}$

$\Rightarrow MR = 5\sqrt{3}$ cm

Also, RN = $5\sqrt{3}$ cm {∵ Tangents from the same external point are congruent to each other.}

(2) $\tan R = \frac{OM}{MR} = \frac{5}{5\sqrt{3}}$

$\Rightarrow \tan R = \frac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow \angle MRO = 30°$

(3) Similarly, $\angle NRO = 30°$

$\Rightarrow \angle MRN = \angle MRO + \angle NRO = 30° + 30° = 60°$

Q. 3. Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects ∠MRN as well as ∠MON.

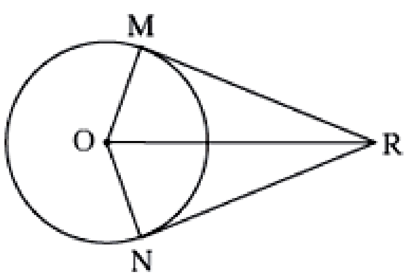


Fig. 3.21

Answer : In triangle MOR and triangle NOR,

MR = NR {∵Tangents from same external point are congruent to each other.}

OR = OR {Common}

OM = ON {Radius of the circle}

$\Rightarrow \Delta MOR \cong \Delta NOR$ {By SSS}

$\Rightarrow \angle ROM = \angle RON$

And ∠MRO = ∠NRO {C.P.C.T.}

Hence proved that seg OR bisects ∠MRNas well as ∠MON.

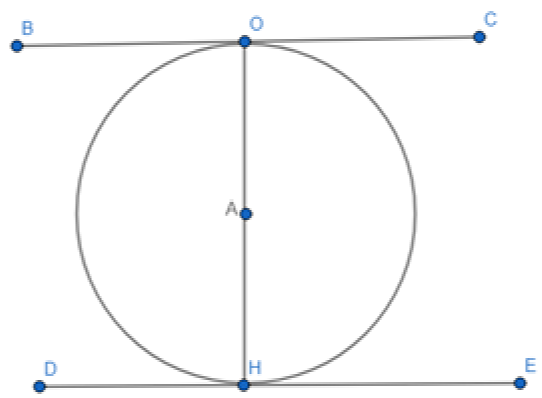
Q. 4. What is the distance between two parallel tangents of a circle having radius4.5 cm ? Justify your answer.

Answer : Let BC and DE be the parallel tangents to a circle centered at A with point of contact O and H respectively. On joining OH, we find OH is the diameter of the circle.∠ BOA = 90° = ∠ DHA {Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.}

Distance between BC and DE = OH

∵ OH is perpendicular to BC and DE.

OH = 2 × 4.5 cm = 9 cm



Practice Set 3.2

Q. 1. Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.

Answer : Given: Two circles are touching each other internally.

∵The distance between the centres of the circles touching internally is equal to the difference of their radii.

⇒ Distance between their centres = 4.8 cm – 3.5 cm = 1.3 cm

Q. 2. Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.

Answer : Given: Two circles are touching each other externally

We know that if the circles touch each other externally, distance between their centres is equal to the sum of their radii.

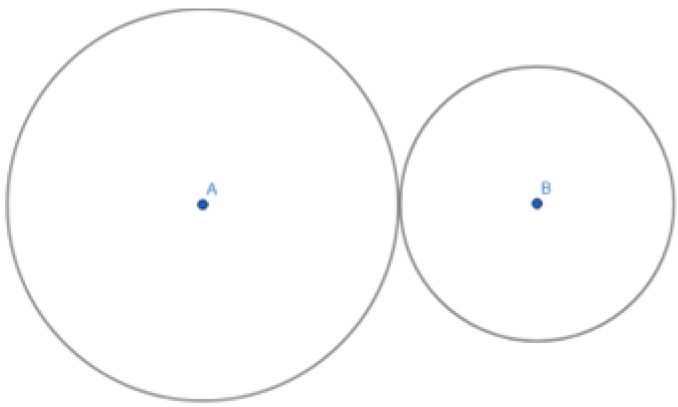
⇒ Distance between their centres = 5.5 cm + 4.2 cm = 9.7 cm

Q. 3. If radii of two circles are 4 cm and 2.8 cm. Draw figure of these circles touching each other –

(i) externally
(ii) internally.

Answer :

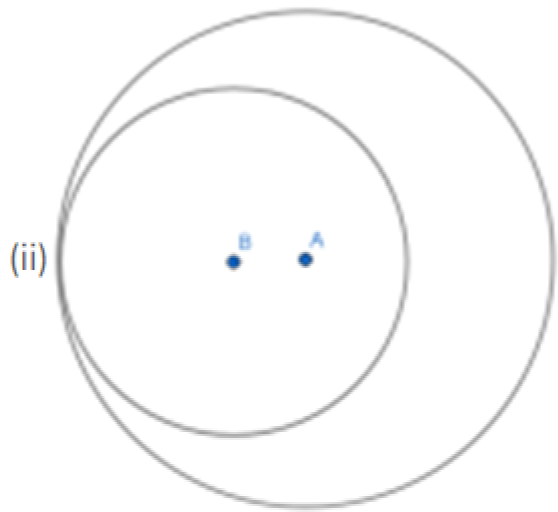
(i)



Steps of construction:

1. Draw a circle with radius 4cm and centre A.

2. Draw another circle with radius 2.8 cm and centre B such that they touch each other externally.



<u>Steps of construction:</u>

1. Draw a circle with radius 4cm and centre A.

2. Draw another circle with radius 2.8 cm and centre B such that they touch each other internally.

Q. 4. In fig 3.27, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively. Prove that -

(1) seg AP || seg BQ,
(2) ∆ APR ~ ∆ RQB, and
(3) Find ∠ RQB if ∠ PAR = 35°

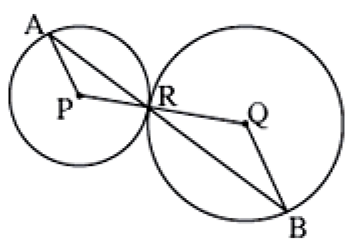


Fig. 3.27

Answer : (1) In ∆APR,

AP = RP {Radius of the circle with centre P}

∠PAR = ∠PRA … (1)

In ∆RQB,

RQ = QB {Radius of the circle with centre Q}

$\angle QRB = \angle QBR$ …. (2)

$\Rightarrow \angle PRA = \angle QRB$ {Vertically Opposite Angle} ….(3)

$\Rightarrow \angle PAR = \angle QBR$ {From (1), (2) and (3)}

$\Rightarrow$ Alternate interior angles are equal.

$\Rightarrow AP \parallel BQ$

Hence, proved.

(2) In Δ APR and Δ RQB,

$\angle PAR = \angle QBR$ and $\angle PRA = \angle QRB$ {From (1) and (2)}

$\Rightarrow \Delta APR \sim \Delta RQB$ {AA}

Hence, proved.

(4) Given: $\angle PAR = 35°$

$\Rightarrow \angle QBR = 35° = \angle QRB$ {Proved previously}

In Δ RQB,

$\Rightarrow \angle RQB + \angle QRB + \angle QBR = 180°$ {Angle sum property of the triangle}

$\Rightarrow \angle RQB + 35° + 35° = 180°$

$\Rightarrow \angle RQB = 180° - 70° = 110°$

Q. 5. In fig 3.28 the circles with centres A and B touch each other at E. Line is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii Fig. 3.28 of the circles are 4 cm, 6 cm.

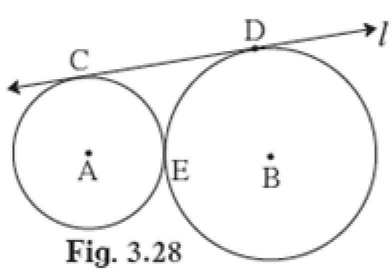
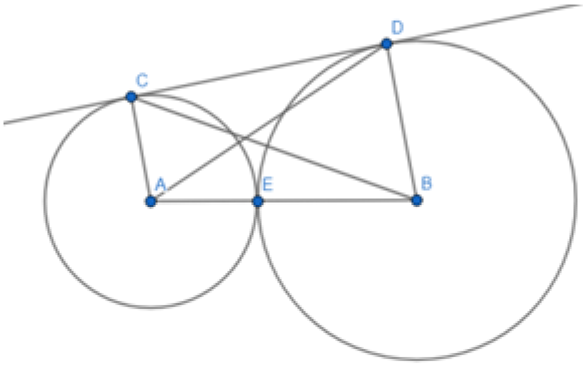


Fig. 3.28

Answer :



Given that two circles with centre A and B touch each other externally. We know that if the circles touch each other externally, distance between their centres is equal to the sum of their radii.

$\Rightarrow$ AB = (4 + 6) cm = 10 cm

In ΔABC right-angles at A,

$BC^2 = CA^2 + AB^2$ {Using Pythagoras theorem}

$\Rightarrow BC^2 = 4^2 + 10^2$

$\Rightarrow BC^2 = 16 + 100$

$\Rightarrow BC = \sqrt{116}$ cm

In ΔDBC,

$\angle BDC = 90°$ because D is the point of contact of tangent CD to circle centred B

$BC^2 = CD^2 + DB^2$ {Using Pythagoras theorem}

$\Rightarrow CD^2 = 116 - 6^2$

$\Rightarrow CD^2 = 116 - 36$

$\Rightarrow CD = \sqrt{80}$ cm $= 4\sqrt{5}$

Practice Set 3.3

Q. 1. In figure 3.37, points G, D, E, F are concyclic points of a circle with centre C.

∠ ECF = 70°, m(arc DGF) = 200° find m(arc DE) and m(arc DEF).

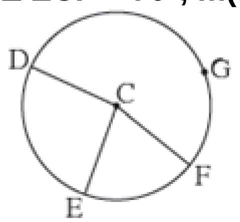


Fig. 3.37

Answer : Given ∠ECF = 70° and m(arc DGF) = 200°

We know that measure of major arc = 360° - measure of minor arc

m(arc DGF) = 360° - m(arc DF)

⇒ m(arc DF) = 360° - 200° = 160°

⇒ ∠ DCF = 160°

∵ The measure of a minor arc is the measure of its central angle.

∴ m(arc DEF) = 160°

So, ∠DCE = ∠ DCF -∠ECF = 160° - 70°

⇒ ∠DCE = 90°

The measure of a minor arc is the measure of its central angle.

m(arc DE) = 90°

Q. 2. In fig 3.38 ∆ QRS is an equilateral triangle. Prove that,

(1) arc RS ≅ arc QS ≅ arc QR
(2) m(arc QRS) = 240°.

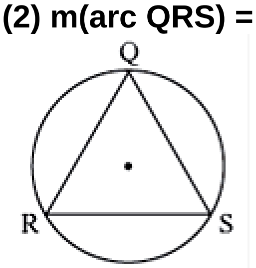


Fig. 3.38

Answer : (1) Two arcs are congruent if their measures and radii are equal.

∵ ∆ QRS is an equilateral triangle

∴ RS = QS = QR

⇒ arc RS ≅ arc QS ≅ arc QR

(2) Let O be the centre of the circle.

m(arc QS) = ∠ QOS

∠ QOS + ∠ QOR + ∠ SOR = 360°

⇒ 3∠ QOS = 360° {∵ ΔQRS is an equilateral triangle}

⇒ ∠ QOS = 120°

m(arc QS) = 120°

m(arc QRS) = 360° - 120° {∵Measure of a major arc = 360°- measure of its corresponding minor arc}

⇒ m(arc QRS) = 240°

Q. 3. In fig 3.39 chord AB ≅ chord CD, Prove that, arc AC ≅ arc BD

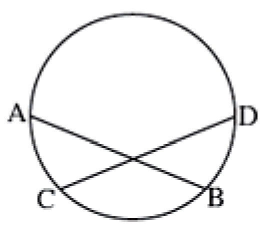


Fig. 3.39

Answer : ∵ Chord AB ≅ chord CD

∴ m(arc AB) = m(arc CD){Corresponding arcs of congruent chords of a circle (or congruent circles) are congruent}

Subtract m(arc CB) from above,

m(arc AB)– m(arc CB) = m(arc CD) – m(arc CB)

⇒ m(arc AC) = m(arc BD)

⇒ arc AC ≅ arc BD

Hence, proved.

Practice Set 3.4

Q. 1. In figure 3.56, in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.

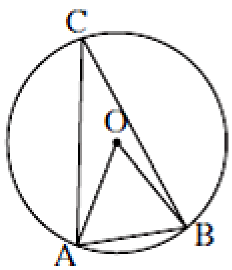


Fig. 3.56

(1) ∠ AOB (2) ∠ ACB
(3) arc AB (4) arc ACB.

Answer : (1) In ΔAOB,

AB = OA = OB = radius of circle

⇒ ΔAOB is an equilateral triangle

∠ AOB + ∠ ABO + ∠ BAO = 180° {Angle sum property}

⇒ 3∠ AOB = 180° {All the angles are equal}

∠ AOB = 60°

(2) ∠ AOB = 2 × ∠ ACB {The measure of an inscribed angle is half the measure of the arc intercepted by it.}

⇒ ∠ ACB = 30°

(3)∠ AOB = 60°

⇒ arc(AB) = 60° {The measure of a minor arc is the measure of its central angle.}

(4) Using Measure of a major arc = 360°- measure of its corresponding minor arc

⇒ arc(ACB) = 360° - arc(AB)

⇒ arc(ACB) = 360° - 60° = 300°

Q. 2. In figure 3.57, □PQRS is cyclic. Side PQ ≅ side RQ. ∠ PSR = 110°, Find-

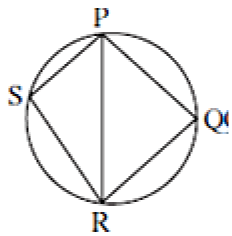


Fig. 3.57

(1) measure of ∠ PQR
(2) m(arc PQR)
(3) m(arc QR)
(4) measure of ∠ PRQ

Answer : (1) Given PQRS is a cyclic quadrilateral.

∵ Opposite angles of a cyclic quadrilateral are supplementary

$\Rightarrow\angle$ PSR + $\angle$ PQR = 180°

$\Rightarrow\angle$ PQR = 180° - 110°

$\Rightarrow\angle$ PQR = 70°

(2) 2 × $\angle$ PQR = m(arc PR){The measure of an inscribed angle is half the measure of the arc intercepted by it.}

m(arc PR) = 140°

$\Rightarrow$ m(arc PQR) = 360° -140° = 220° {Using Measure of a major arc = 360°- measure of its corresponding minor arc}

(3) Side PQ ≅ side RQ

∴ m(arc PQ) = m(arc RQ){Corresponding arcs of congruent chords of a circle (or congruent circles) are congruent}

$\Rightarrow$ m(arc PQR) = m(arc PQ) + m(arc RQ)

$\Rightarrow$ m(arc PQR) = 2 × m(arc PQ)

$\Rightarrow$ m(arc PQ) = 110°

(4) In Δ PQR,

$\angle$ PQR + $\angle$ QRP + $\angle$ RPQ = 180° {Angle sum property}

$\Rightarrow \angle PRQ + \angle RPQ = 180° - \angle PQR$

$\Rightarrow 2\angle PRQ = 180° - 70°$ {∵ side PQ ≅ side RQ}

$\Rightarrow \angle PRQ = 55°$

Q. 3. □MRPN is cyclic, ∠R = (5x - 13)°, ∠N = (4x + 4)°. Find measures of $\sqrt{R}$ and $\sqrt{N}$.

Answer : Given MRPN is a cyclic quadrilateral.

$\Rightarrow \angle R + \angle N = 180°$ {Using Opposite angles of a cyclic quadrilateral are supplementary}

$\Rightarrow (5x - 13)° + (4x + 4)° = 180°$

$\Rightarrow 9x – 9 = 180°$

$\Rightarrow x – 1 = 20°$

$\Rightarrow x = 21°$

$\angle R = (5x - 13)° = 5 \times 21 – 13 = 105 – 13 = 92°$

$\angle N = (4x + 4)° = 4 \times 21 + 4 = 84 + 4 = 88°$

Q. 4. In figure 3.58, seg RS is a diameter of the circle with centre O. Point T lies in the exterior of the circle. Prove that ∠ RTS is an acute angle.

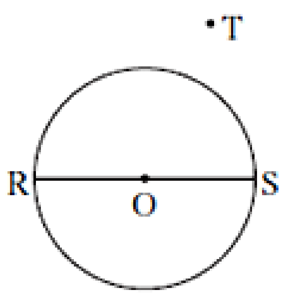
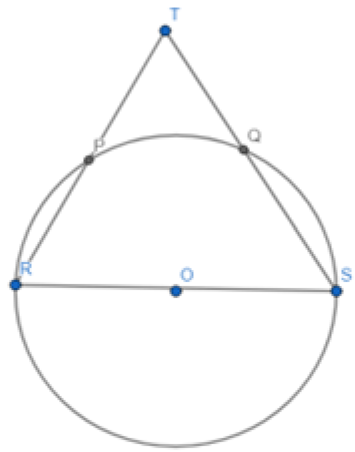


Fig. 3.58

Answer :



Given RS is the diameter

⇒ ∠ ROS = 180°

m(arc RS) = 180°

Now, ∠ RTS is an external angle.

$$\angle RTS = \frac{1}{2}[m(arcRS) - m(arcPQ)]$$

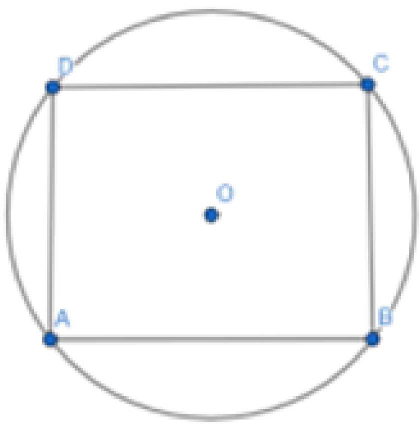
$$\Rightarrow \angle RTS = \frac{1}{2}[180° - m(arcPQ)]$$

$$\Rightarrow \angle RTS = 90° - \frac{1}{2}m(arcPQ)$$

Hence, ∠ RTS is an acute angle.

Q. 5. Prove that, any rectangle is a cyclic quadrilateral.

Answer :



In ABCD,

∠A = 90°{∵ angle of a rectangle is 90°.}

∠C = 90° {opposite angles are equals}

⇒ ∠ A + ∠ C = 180°

If opposite angles are supplementary, the quadrilateral is cyclic.

∴ ABCD is cyclic.

Q. 6. In figure 3.59, altitudes YZ and XT of

∆ WXY intersect at P. Prove that,

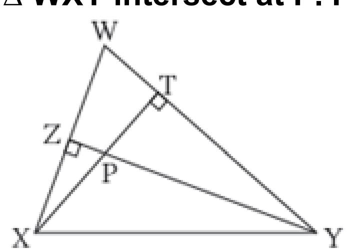


Fig. 3.59

(1) □ WZPT is cyclic.
(2) Points X, Z, T, Y are concyclic.

Answer : (1)In WZPT,

∠ WZP = ∠ WTP = 90° {YZ and XT are the altitudes}

If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

⇒ WZPT is cyclic.

(2) ∵ X, Z, T, Y lie on same circle, ∴ they are concyclic.

Q. 7. In figure 3.60, m(arc NS) = 125°, m(arc EF) = 37°, find the measure ∠ NMS.

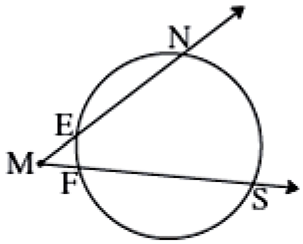


Fig. 3.60

Answer : Given m(arc NS) = 125°, m(arc EF) = 37°

Also, ∠ NMS is an external angle, so

$$\angle NMS = \frac{1}{2}[m(arcNS) - m(arcEF)]$$

$$\Rightarrow \angle NMS = \frac{1}{2}[125° - 37°]$$

$$\Rightarrow \angle NMS = \frac{1}{2} \times 88° = 44°$$

Q. 8. In figure 3.61, chords AC and DE intersect at B. If ∠ ABE = 108°, m(arc AE) = 95°, find m(arc DC).

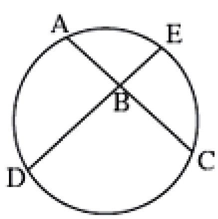


Fig. 3.61

Answer : Given ∠ ABE = 108°, m(arc AE) = 95°

Using the property of the secant,

$$\angle ABE = \frac{1}{2}[m(arc\,AE) + m(arc\,DC)]$$

$$\Rightarrow 108° = \frac{1}{2}[95° + m(arc\,DC)]$$

⇒ m(arc DC) = 108° × 2 – 95°

⇒ m(arc DC) = 121°

Practice Set 3.5

Q. 1. In figure 3.77, ray PQ touches the circle at point Q. PQ = 12, PR = 8, find PS and RS.

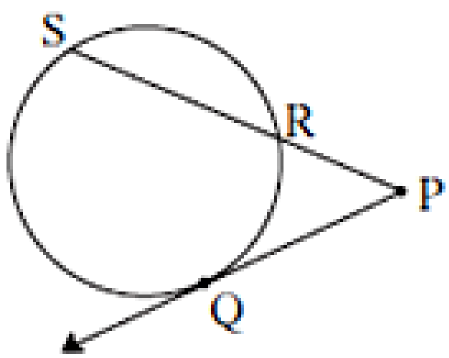


Fig. 3.77

Answer : Given PQ = 12, PR = 8

SP × RP = PQ^2

This property is known as tangent secant segments theorem.

⇒ PS × 8 = 12^2

$$\Rightarrow PS = \frac{144}{8} = 18$$

RS = PS – RP = 18 – 8 = 10

Q. 2. In figure 3.78, chord MN and chord RS intersect at point D.

(1) If RD = 15, DS = 4, MD = 8 find DN

(2) If RS = 18, MD = 9, DN = 8 find DS

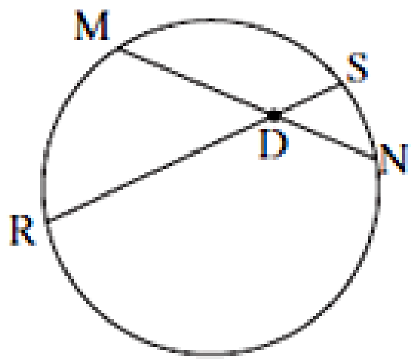


Fig. 3.78

Answer : (1) Given RD = 15, DS = 4, MD = 8

MD × DN = RD × DS

This property is known as theorem of chords intersecting inside the circle.

⇒ 8 × DN = 15 × 4

$$\Rightarrow DN = \frac{15}{2} = 7.5$$

(2) Given RS = 18, MD = 9, DN = 8

Here, RS = 18

Let RD = x and DS = 18 – x

MD × DN = RD × DS

This property is known as theorem of chords intersecting inside the circle.

⇒ 8 × 9 = x × (18 – x)

⇒ $18x - x^2 = 72$

⇒ $x^2 - 18x + 72 = 0$

⇒ (x – 12)(x – 6) = 0

⇒ x = 12 or 6

⇒ DS = 6 or 12

Q. 3. In figure 3.79, O is the centre of the circle and B is a point of contact. seg OE $\perp$ seg AD, AB = 12, AC = 8, find

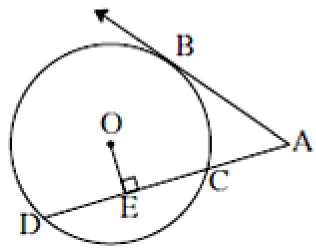


Fig. 3.79

(1) AD
(2) DC
(3) DE.

Answer : (1)Given: OE $\perp$ AD, AB = 12, AC = 8

$\Rightarrow$ AD × AC = AB^2

This property is known as tangent secant segments theorem.

$\Rightarrow$ AD × 8 = 12^2

$$\Rightarrow AD = \frac{144}{8} = 18$$

(2) DC = AD – AC = 18 – 8 = 10

(3) As we know that a perpendicular from centre divides the chord in two equal parts. Here, OE $\perp$ AD.

$\Rightarrow$ DE = EC

$\Rightarrow$ DE + EC = DC

$\Rightarrow$ 2DE = DC

$$\Rightarrow DE = \frac{1}{2}DC = 5$$

Q. 4. In figure 3.80, if PQ = 6, QR = 10, PS = 8 find TS.

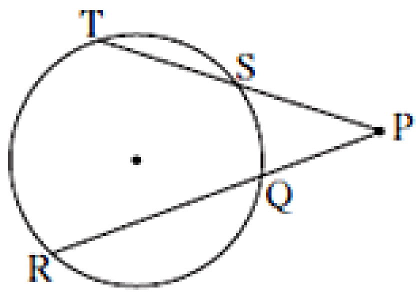


Fig. 3.80

Answer : Given: PQ = 6, QR = 10, PS = 8

PT × PS = PR × PQ

This property is known as theorem of chords intersecting outside the circle.

⇒ PR = PQ + RQ = 6 + 10 = 16

⇒ PT × 8 = 16 × 6

⇒ PT = 12

TS = PT – PS = 12 – 8 = 4

Q. 5. In figure 3.81, seg EF is a diameter and seg DF is a tangent segment. The radius of the circle is r. Prove that, DE × GE = $4r^2$

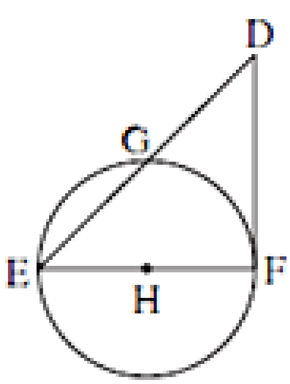


Fig. 3.81

Answer : In ∆DEF,

∠DFE = 90° {Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.}

Given: EF = diameter of the circle.

$DE^2 = DF^2 + EF^2$ {Using Pythagoras theorem}

$\Rightarrow DE^2 = DF^2 + (2r)^2$

$\Rightarrow DE^2 = DF^2 + 4r^2$

$\Rightarrow DF^2 = DE^2 - 4r^2$

Also, $DE \times DG = DF^2$

This property is known as tangent secant segments theorem.

$\Rightarrow DE \times DG = DE^2 - 4r^2$

$\Rightarrow DE^2 - DE \times DG = 4r^2$

$\Rightarrow DE(DE - DG) = 4r^2$

$\Rightarrow DE \times EG = 4r^2$

Hence, proved.

Problem Set 3

Q. 1. A. Four alternative answers for each of the following questions are given. Choose the correct alternative.

Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centers?
A. 4.4 cm

B. 8.8 cm
C. 2.2 cm
D. 8.8 or 2.2 cm

Answer : Given that both the circles touch each other but not specified externally or internally.

The distance between the centres of the circles touching internally is equal to the difference of their radii.

$\Rightarrow$ Distance between their centres = 5.5 cm – 3.3 cm = 2.2 cm

If the circles touch each other externally, distance between their centres is equal to the sum of their radii.

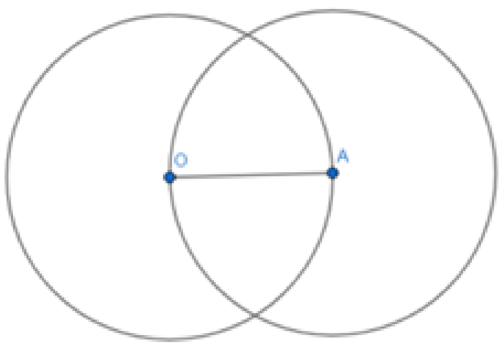
⇒ Distance between their centres = 5.5 cm + 3.3 cm = 8.8 cm

Q. 1. B. Four alternative answers for each of the following questions are given. Choose the correct alternative.

Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle?
A. 6 cm

B. 12 cm
C. 24 cm
D. can't say

Answer : Given OA = 12



From the figure, OA is the radius of both the circles.

Given that distance between their centres is OA = 12

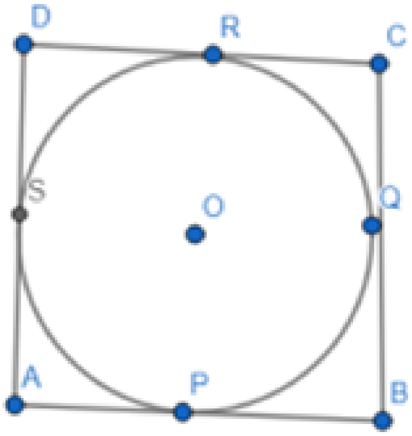
∴ Radius of the circles = 12

Q. 1. C. Four alternative answers for each of the following questions are given. Choose the correct alternative.

A circle touches all sides of a parallelogram. So the parallelogram must be a,.................. .
A. rectangle

B. rhombus
C. square
D. trapezium

Answer :



Let ABCD be a parallelogram which circumscribes the circle.

AP = AS [Tangents drawn from an external point to a circle are equal in length]

BP = BQ [Tangents drawn from an external point to a circle are equal in length]

CR = CQ [Tangents drawn from an external point to a circle are equal in length]

DR = DS [Tangents drawn from an external point to a circle are equal in length]

Consider, (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

But AB = CD and BC = AD [Opposite sides of parallelogram ABCD]

AB + CD = AD + BC

Hence 2AB = 2BC

Therefore, AB = BC

Similarly, we get AB = DA and DA = CD

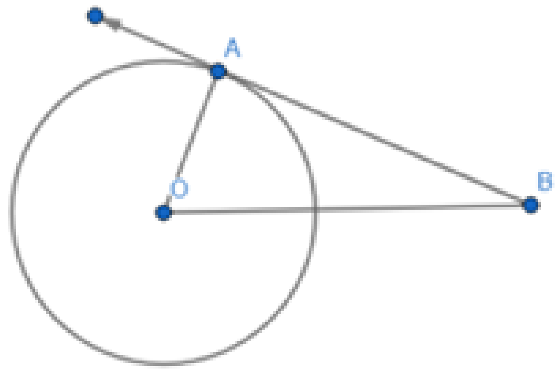
Thus, ABCD is a rhombus.

Q. 1. D. Four alternative answers for each of the following questions are given. Choose the correct alternative.

Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.
A. 25 cm

B. 24 cm
C. 7 cm
D. 14 cm

Answer :



Given: BO = 12.5 cm and AB = 12cm

In $\triangle AOB$,

$\angle OAB = 90°$ {Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.}

$BO^2 = AB^2 + OA^2$ {Using Pythagoras theorem}

$\Rightarrow (12.5)^2 = 12^2 + OA^2$

$\Rightarrow OA^2 = 156.25 - 144$

$\Rightarrow OA = \sqrt{12.25}$

$\Rightarrow OA = 3.5cm$

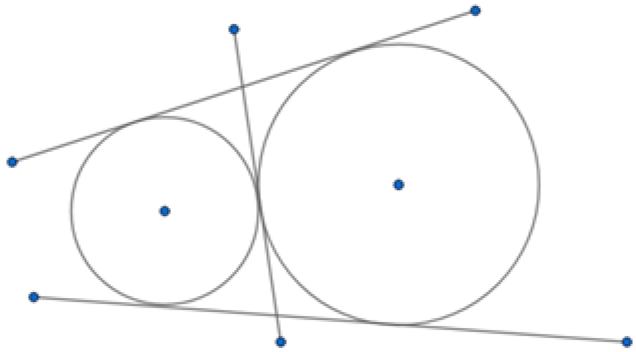
Radius = 3.5 cm

$\Rightarrow$ Diameter = 7 cm

Q. 1. E. Four alternative answers for each of the following questions are given. Choose the correct alternative.
If two circles are touching externally, how many common tangents of them can be drawn?
A. One
B. Two

C. Three
D. Four

Answer :



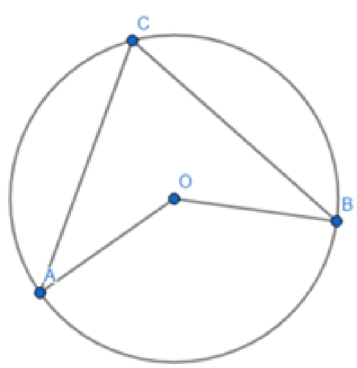
If two circles are touching each other externally, they have 3 tangents in common. The above figure proves this statement.

There are three common tangents for the given two circles.

Q. 1. F. Four alternative answers for each of the following questions are given. Choose the correct alternative.

∠ ACB is inscribed in arc ACB of a circle with centre O. If ∠ ACB = 65°,find m(arc ACB).
A. 65°
B. 130°
C. 295°
D. 230°

Answer :



Given ∠ACB = 65°

⇒ ∠ AOB = 2 × 65° = 130° {∵ The measure of an inscribed angle is half the measure of the arc intercepted by it.}

m(AB) = 130°

So, m(arc ACB) = 360° - m(AB) {∵ Measure of a major arc = 360°- measure of its corresponding minor arc}

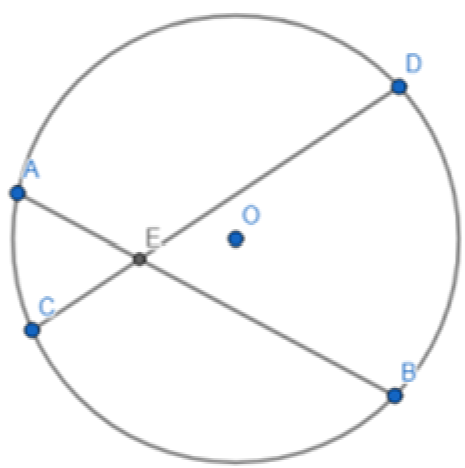
⇒ m(arc ACB) = 360° - 130° = 230°

Q. 1. G. Four alternative answers for each of the following questions are given. Choose the correct alternative.

Chords AB and CD of a circle intersect inside the circle at point E. If AE = 5.6,EB = 10, CE = 8, find ED.
A. 7

B. 8
C. 11.2
D. 9

Answer :



Given: AE = 5.6,EB = 10, CE = 8

We know that AE × EB = CE × ED

This property is known as theorem of chords intersecting inside the circle.

⇒ 5.6 × 10 = 8 × ED

⇒ ED = 7

Q. 1. H. Four alternative answers for each of the following questions are given. Choose the correct alternative.

In a cyclic □ ABCD, twice the measure of $\angle A$ is thrice the measure of $\angle C$. Find the measure of $\angle C$?
A. 36
B. 72
C. 90
D. 108

Answer : Given that 2∠ A = 3∠ C

We know that in a cyclic quadrilateral opposite angles are supplementary to each other.

⇒∠ A + ∠ C = 180°

$$\Rightarrow \frac{3}{2}\angle C + \angle C = 180°$$

$$\Rightarrow \frac{5}{2}\angle C = 180°$$

⇒∠ C = 72°

Q. 1. I. Four alternative answers for each of the following questions are given. Choose the correct alternative.

Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. Nopoint, except point B, is common to the arcs. Which is the type of Δ ABC?
A. Equilateral triangle
B. Scalene triangle
C. Right angled triangle
D. Isosceles triangle

Answer : Angle subtended by the arcs at centre = 120°

⇒ Angle subtended by the arc at the remaining part of the circle = 60° {The measure of an inscribed angle is half the measure of the arc intercepted by it.}

∵ Interior angles of the triangle ABC = 60°

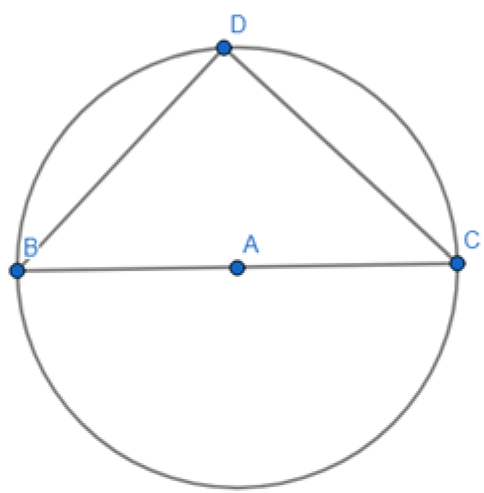
∴ It is an equilateral triangle.

Q. 1. J. Four alternative answers for each of the following questions are given. Choose the correct alternative.

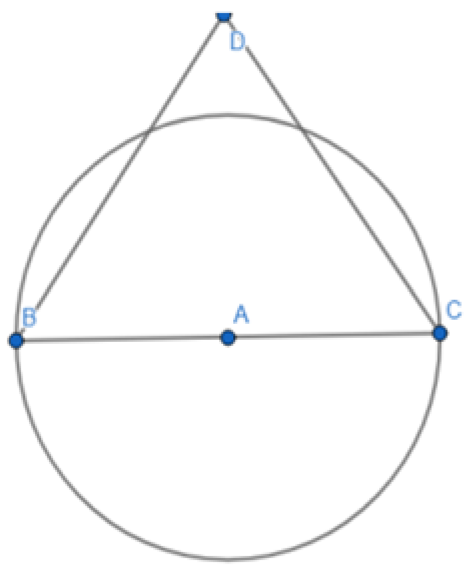
Seg XZ is a diameter of a circle. Point Y lies in its interior. How many of the following statements are true?
(i) It is not possible that ∠XYZ is an acute angle.
(ii) ∠ XYZ can't be a right angle.
(iii) ∠ XYZ is an obtuse angle.
(iv) Can't make a definite statement for measure of ∠ XYZ.

A. Only one
B. Only two
C. Only three
D. All

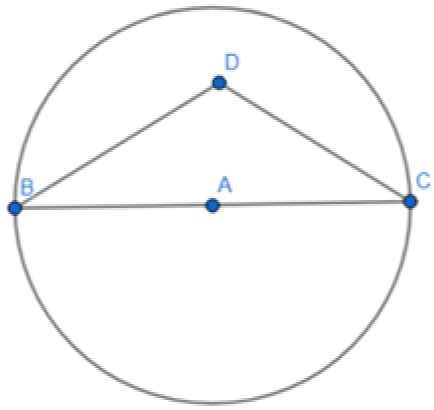
Answer : If Y would have lied on circumference ∠ XYZ = 90° since XZ is the diameter.



If Y lied outside the circle, ∠ XYZ = acute angle



∴ ∠ XYZ is an obtuse angle.



Statements (i), (ii) and (iii) are true.

Q. 2. Line l touches a circle with centre Oat point P. If radius of the circle is 9 cm,answer the following.

(1) What is d(O, P) = ? Why ?
(2) If d(O, Q) = 8 cm, where does the point Q lie?
(3) If d(PQ) = 15 cm, How many locations of point R are line online l ? At what distance will each of them be from point P?

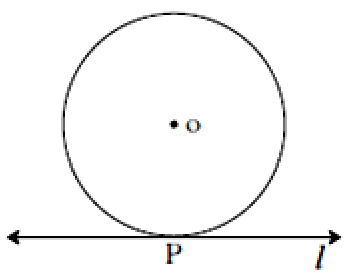


Fig. 3.82

Answer : (1)The perpendicular distance of O from P = radius of the circle = 9 cm.

(2) Q lies in the interior of the circle because P lieing on the circumference of the circle is at a distance of 9 cm.

(3) Position of R is not specified.

Q. 3. In figure 3.83, M is the centre of the circle and seg KL is a tangent segment.

If MK = 12, KL = $6\sqrt{3}$ **then find -**
(1) Radius of the circle.

(2) Measures of ∠ K and ∠ M.

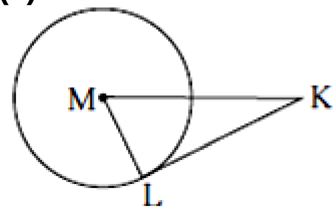


Fig. 3.83

Answer : (1) Here LM is the radius of the circle

⇒ ∠KML = 90° Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.

In triangle MLK right-angled at L,

Given MK = 12, KL = 6√3,

$MK^2 = LM^2 + KL^2$ {Using Pythagoras theorem}

⇒ $LM^2 = 12^2 - 6\sqrt{3}^2$

⇒ $LM^2 = 144 - 108$

⇒ LM = √36

⇒ LM = 6 cm

(2) $\tan K = \frac{ML}{LK}$

$$\Rightarrow \tan K = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30°$$

⇒∠K = 30°

∠ M + ∠ K + ∠ L = 180° {Angle sum property of the triangle}

So, ∠ M = 180° - ∠ K - ∠ L

⇒ ∠ M = 180° - 30° - 90° = 60°

Q. 4. In figure 3.84, O is the centre of the circle. Seg AB, seg AC are tangent segments. Radius of the circle is r and l(AB) = r , Prove that, □ABOC is a square.

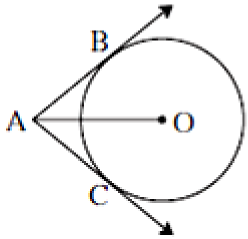


Fig. 3.84

Answer : Given: AB = r = radius of the circle

Here, AB = AC = r {tangents from the same external point are equal}

And OB = OC = r = radius of the circle.

⇒ ∠ OBA = ∠ OCA = 90° Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.

∵ Sides of ABOC are equal and opposite angles are 90° each

Hence, ABOC is a square.

Q. 5. In figure 3.85, □ABOC is a parallelogram. It circumscribes the circle with cnetre T. Point E,F, G, H are touching points. If AE = 4.5,EB = 5.5, find AD.

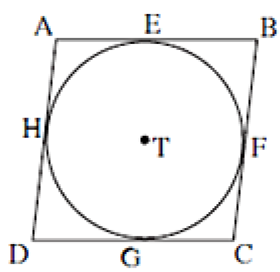


Fig. 3.85

Answer : Given: AE = 4.5, EB = 5.5

Here, AE = AH = 4.5 {tangents from same external point are equal}

EB = BF = 5.5{tangents from same external point are equal}

∵ Opposite sides of a parallelogram are equal

∴ AE = DG and EB = GC

Also, DH = DG = 4.5{tangents from same external point are equal}

And FC = GC = 5.5 {tangents from same external point are equal}

⇒ AD = AH + HD = 10

Q. 6. In figure 3.86, circle with centre M touches the circle with centre N at point T. Radius RM touches the smaller circle at S. Radii of circles are 9 cm and 2.5 cm. Find the answers to the following questions hence find the ratio MS:SR.

(1) Find the length of segment MT
(2) Find the length of seg MN
(3) Find the measure of ∠ NSM.

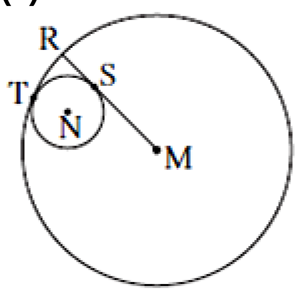


Fig. 3.86

Answer : (1)MT = radius of the big circle = 9 cm

(2) MN = MT – TN = 9 – 2.5 = 6.5 cm

(3) SM is the tangent to the circle with radius 2.5 cm with S being point of contact.

∠ NSM = 90° Using tangent-radius theorem which states that a tangent at any point of a circle is perpendicular to the radius at the point of contact.

In ∆MSN,

∠ MSN = 90°{∵ MS is the tangent to the small circle with point of contact S}

⇒ $MN^2 = MS^2 + NS^2$

$MS^2 = MN^2 – NS^2$

⇒ $MS^2 = 6.5^2 – 2.5^2$

⇒ $MS^2 = 36$

⇒ MS = 6 cm

Now, SR = MR – MS = 9 – 6 = 3 cm

⇒ MS: SR = 6:3 = 2:1

Q. 7. In the adjoining figure circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively. Prove that, radius XA || radius YB. Fill in the blanks and complete the proof.

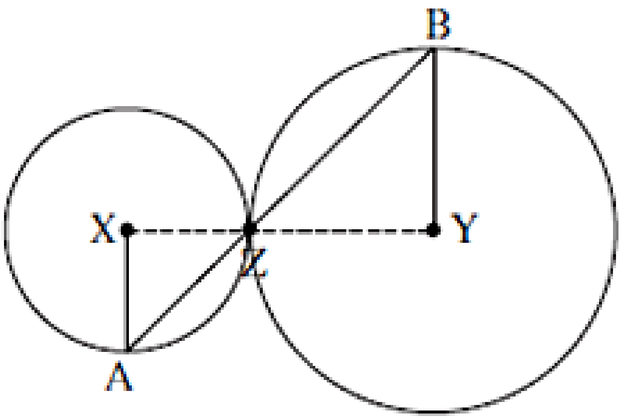


Fig. 3.87

Construction: Draw segments XZ and ..YZ........ .
Proof: By theorem of touching circles, points X, Z, Y are ..concyclic........ .
∴ $\angle XZA \cong$ ..∠ YZB ....vertically opposite angles
Let $\angle XZA = \angle BZY = a$ ..... (I)
Now, seg XA ≅seg XZ .... (...radius of the same circle.......)
∴ $\angle XAZ$ = ...∠ XZA...... = a .... (isosceles triangle theorem) (II)
similarly, seg YB ≅ .YZ......... .... (.radius of the same circle.........)
∴ $\angle BZY$ =.∠ZBY......... = a .... (.isosceles triangle theorem.........) (III)
∴from (I), (II), (III),
∠ XAZ = .∠ ZBY.........
∴radius XA || radius YB .... (..since alternate interior angles are equal........)

Answer : Construction: Draw segments XZ and YZ.

Proof: By theorem of touching circles, points X, Z, Y are concyclic.

∠ XZA = ∠ YZB {vertically opposite angles}

Let ∠ XZA = ∠ BZY = a (I)

Now, seg XA ≅segXZ (radius of the same circle)

∵ ∠ XAZ = ∠ XZA = a (isosceles triangle theorem) (II)

Similarly, seg YB ≅ YZ (radius of the same circle)

∴ ∠ BZY = ∠ZBY = a (isosceles triangle theorem) (III)

∴from (I), (II), (III),

∠ XAZ = ∠ ZBY

∴ Radius XA || radius YB (since alternate interior angles are equal)

Q. 8. In figure 3.88, circles with centres X and Y touch internally at point Z. Seg BZ is a chord of bigger circle and intersects smaller circle at point A. Prove that, seg AX || seg BY.

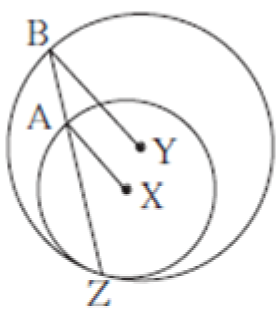


Fig. 3.88

Answer : XA and YB are the radii of the respective circles.

AZ and BZ are the chords of the circles.

In triangle XAZ,

AX = XZ {Radii of the same circle}

⇒ ∠ XAZ = ∠ XZA {angles opposite to equal sides are equal}

In triangle YBZ,

YB = YZ {Radii of the same circle}

⇒ ∠ YBZ = ∠ YZB {angles opposite to equal sides are equal}

⇒ ∠ XAZ = ∠ XZA = ∠ YBZ = ∠ YZB

∵ Corresponding angles are equal

⇒ XA||YB

Q. 9. In figure 3.89, line l touches the circle with centre O at point P. Q is the mid point of radius OP. RS is a chord through Q such that chords RS || line l. If RS = 12 find the radius of the circle.

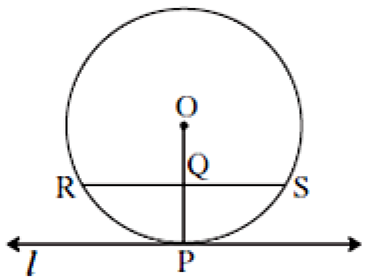


Fig. 3.89

Answer : The radius of the circle will bisect the chord RS. Therefore, RQ = QS = 1/2 × 12 = 6

Let the radius of circle be r,

Now, in Δ OQS, we have,

RQ = 6

OR = r

OQ = 1/2 r

Applying Pythagoras theorem, we get,

$$r^2 = \left(\frac{r}{2}\right)^2 + (6)^2$$

$$r^2 - \frac{r^2}{4} = 36$$

$$\frac{3r^2}{4} = 36$$

$3r^2 = 4 \times 36$

$r^2 = 4 \times 12 = 48$

r = √48 units

Q. 10. In figure 3.90, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, which touches the circle at point T. seg AP ⊥ line PQ and seg BQ ⊥ line PQ. Prove that, seg CP ≅seg CQ.

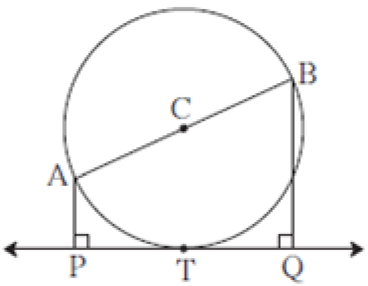
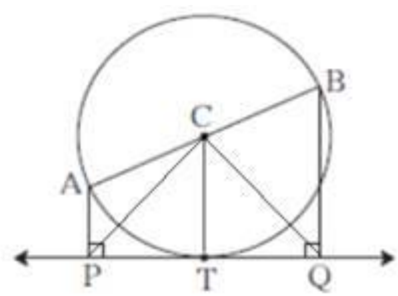


Fig. 3.90

Answer : To Prove: seg CP ≅seg CQ

Construction: Join CP, CQ and CT

Figure:



Since PQ is a tangent to the circle, ∠CTP = ∠CTQ = 90

Since ∠APT = ∠CTP = 90

AP || CT.

Similarly,

CT || BQ.

So, we can say that,

AP || CT || BQ

AB is a line cutting all three parallel lines.

AC = CB (Radius of the circle, and AB is diameter, C is center)

Since C is center point of line AB cutting parallel lines.

We can say these parallel lines are equal distance.

Therefore, PT = TQ.

Now in ΔCTP and ΔCTQ,

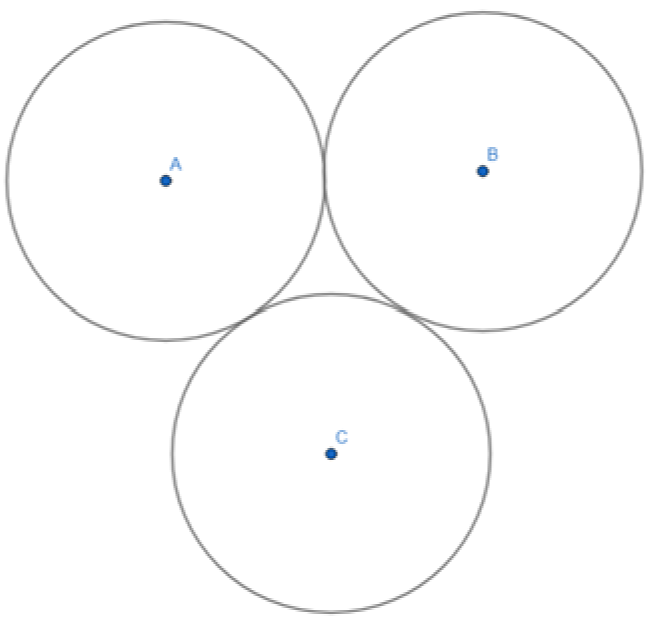
CT is a common side, PT = TQ and ∠CTP = ∠CTQ = 90

CP = CQ. (Pythagoras theorem or congruent triangle theorem)

Hence, Proved.

Q. 11. Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.

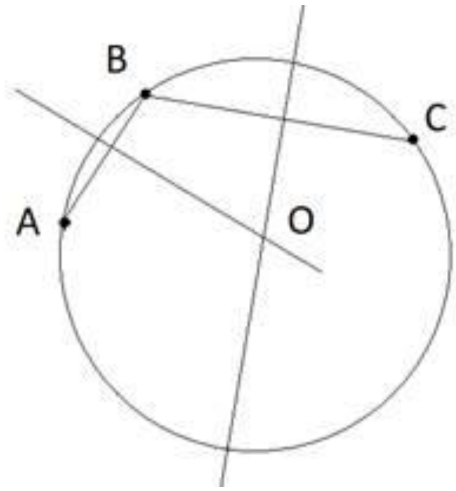
Answer :



Draw a circle with radius 3 and centred at A. Similarly draw other two circles with centre B and C and same radius touching each other externally.

Q. 12. Prove that any three points on a circle cannot be collinear.

Answer :



We draw a circle of any radius and take any three points A, B and C on the circle.

We join A to B and B to C.

We draw perpendicular bisectors of AB and BC.

We know that perpendicular from the center bisects the chord.

Hence the center lies on both of the perpendicular bisectors.

The point where they intersect is the center of the circle.

The perpendiculars of the line segments drawn by joining collinear points is always parallel whereas in circle any three point's perpendicular bisector will always intersect at the center.

Hence, any three points on the circle cannot be collinear.

Q. 13. In figure 3.91, line PR touches the circle at point Q. Answer the following questions with the help of the figure.

(1) What is the sum of ∠ TAQ and∠ TSQ?
(2) Find the angles which are congruent to ∠ AQP.
(3) Which angles are congruent to ∠QTS ?
(4) ∠ TAS = 65°, find the measure of ∠TQS and arc TS.

(5) If ∠AQP = 42°and ∠SQR = 58° find measure of ∠ATS.

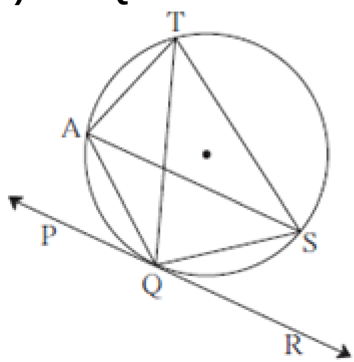


Fig. 3.91

Answer : (1) As TAQS is a cyclic quadrilateral,

∠TAQ + ∠TSQ = 180° (Sum of opposite angles of a cyclic quadrilateral is 180°)

(2) ∠ASQ and ∠ATQ

(3) ∠ QAS and ∠SQR

(4) ∠TAS = 65°

∠ TQS = ∠ TAS = 65° (angle by same arc TS in the same sector)

m(arc TS) = ∠TQS + ∠TAS

⇒ m(arc TS) = 65 + 65 = 130°

(5) ∠AQP + ∠AQS + ∠SQR = 180°

⇒ 42 + ∠AQS + 58 = 180

⇒ ∠AQS + 100 = 180

⇒ ∠AQS = 80

∠ AQS + ∠ ATS = 180° (opposite angles of a cyclic quadrilateral)

⇒ 80 + ∠ATS = 180

⇒ ∠ATS = 100°

Q. 14. In figure 3.92, O is the centre of a circle, chord PQ ≅ chord RS If∠ POR = 70°and (arc RS) = 80°, find -
(1) m(arc PR)

(2) m(arc QS)
(3) m(arc QSR)

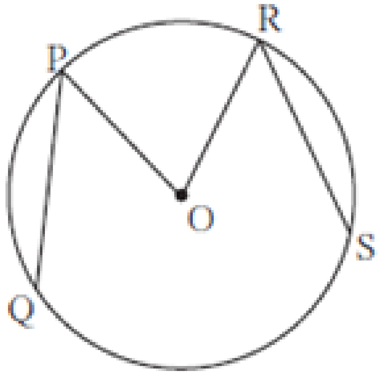


Fig. 3.92

Answer : (1) m(arc PR) = ∠POR = 70°

(2) ∠POQ + ∠ QOS + ∠ROS + ∠POR = 360°

As PQ = RS, ∠ POQ = ∠ROS = 80°

⇒ ∠POQ + ∠ QOS + ∠ROS + ∠POR = 360°

⇒ 80 + ∠QOS + 80 + ∠ 70 = 360

⇒ 230 + ∠ QOS = 360

⇒ ∠QOS = 130°

m(arc QS) = ∠QOS = 130°

(3) m(arc QSR) = ∠QOS + ∠ROS = 130 + 80 = 210°

Q. 15. In figure 3.93, m(arc WY) = 44°,m(arc ZX) = 68°, then

(1) Find the measure of ∠ ZTX.
(2) If WT = 4.8, TX = 8.0, YT = 6.4, find TZ.

(3) If WX = 25, YT = 8, YZ = 26, find WT.

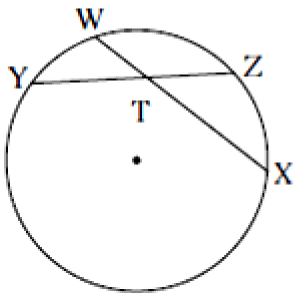


Fig. 3.93

Answer : (1)Given: m(arc WY) = 44°,m(arc ZX) = 68°

We know that

$$\angle ZTX = \frac{1}{2}[m(arcZX) + m(arcWX)]$$

$$\Rightarrow \angle ZTX = \frac{1}{2}(44° + 68°) = 56°$$

(2)Given: WT = 4.8, TX = 8.0, YT = 6.4

We know that WT × TX = YT × TZ {Using secant-tangent theorem}

⇒ 6.4 × TZ = 4.8 × 8

⇒ TZ = 6

(3)Given: WX = 25, YT = 8, YZ = 26

Let WT = x and TX = 25-x

WT × TX = YT × TZ

⇒ x(25-x) = 8 × 26

⇒ (x – 16)(x-9) = 0

⇒ WT = 16 or 9

Q. 16. In figure 3.94,

(1) m(arc CE) = 54°,

m(arc BD) = 23°, find measure of ∠CAE.
(2) If AB = 4.2, BC = 5.4,
AE = 12.0, find AD
(3) If AB = 3.6, AC = 9.0,
AD = 5.4, find AE

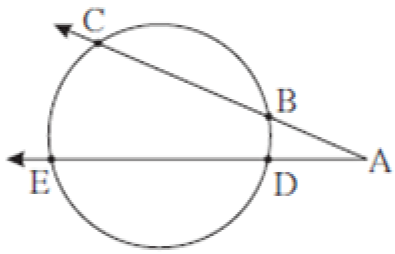


Fig. 3.94

Answer : (1)Given: m(arc CE) = 54°,

m(arc BD) = 23°

∠ CAE is an external angle.

$$\angle CAE = \frac{1}{2}[m(arcCE) - m(arcBD)]$$

$$\angle CAE = \frac{1}{2}[54° - 23°] = 15.5°$$

(2)Given: AB = 4.2, BC = 5.4,AE = 12.0

Here, AB × AC = AD × EA

⇒ AD × 12 = 4.2 × 5.4

⇒ AD = 3.36

(3) Given AB = 3.6, AC = 9.0,

AD = 5.4

Here, AB × AC = AD × EA

⇒ AE × 5.4 = 3.6 × 9

⇒ AE = 6

Q. 17. In figure 3.95, chord EF || chord GH. Prove that, chord EG ≅ chord FH.

Fill in the blanks and write the proof.
Proof : Draw seg GF.

∠ EFG = ∠ FGH ....Alternate interior angles....... $\boxed{}$ (I)
∠ EFG = ...90° ....{inscribed angle theorem}(II)
∠ FGH = ... 90° ....{inscribed angle theorem} (III)
∴m(arc EG) = ...90°............. from (I), (II), (III).
chord EG ≅ chord FH .... $\boxed{}$

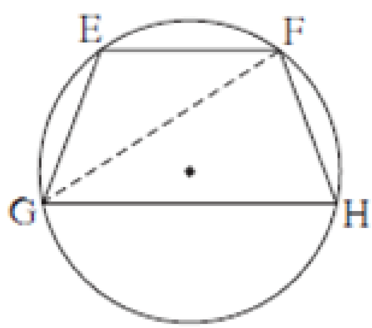


Fig. 3.95

Answer : Proof : Draw seg GF.

∠ EFG = ∠ FGH {Alternate interior angles} (I)

∠ EFG = 90°{inscribed angle theorem}(II)

∠ FGH = 90°{inscribed angle theorem} (III)

∴m(arc EG) = 90° from (I), (II), (III).

Chord EG ≅ chord FH {Corresponding chords of congruent arcs of a circle (or congruent circles) are congruent}

Q. 18. In figure 3.96 P is the point of contact.

(1) If m(arc PR) = 140°,∠ POR = 36°,find m(arc PQ)
(2) If OP = 7.2, OQ = 3.2,find OR and QR

(3) If OP = 7.2, OR = 16.2,find QR.

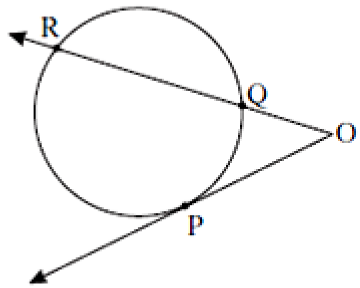


Fig. 3.96

Answer : (1) Given: m(arc PR) = 140°,∠ POR = 36°

∠ ROP is an external angle.

$$\angle ROP = \frac{1}{2}[m(arcRP) - m(arcPQ)]$$

⇒ m(arc PQ) = 140° -2 × 36°

⇒ m(arc PQ) = 68°

(2) Given: OP = 7.2, OQ = 3.2

Here, RO × OQ = OP^2

⇒ RO × 3.2 = 7.2 × 7.2

⇒ RO = 16.2

QR = RO – OQ = 16.2 – 3.2 = 13

(3) Given: OP = 7.2, OR = 16.2

Here, RO × OQ = OP^2

⇒ 16.2 × OQ = 7.2 × 7.2

⇒ OQ = 3.2

QR = RO – OQ = 16.2 – 3.2 = 13

Q. 19. In figure 3.97, circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA ≅seg AB.

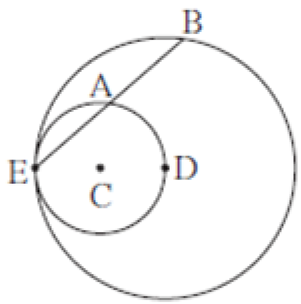
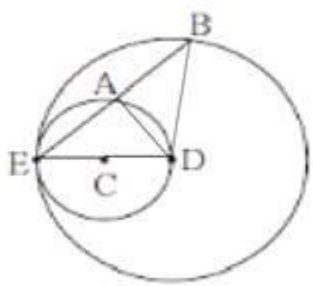


Fig. 3.97

Answer :



We see that the line joining D to E passes through C.

In the smaller circle,

A lies in the semicircle,

∴ ∠EAD = 90°

⇒ DA is perpendicular on the chord EB of the bigger circle.

We know that perpendicular from the center bisects the chord.

Therefore, EA = AB.

Q. 20. In figure 3.98, seg AB is a diameter of a circle with centre O . The bisector of∠ ACB intersects the circle at point D. Prove that, seg AD ≅seg BD.

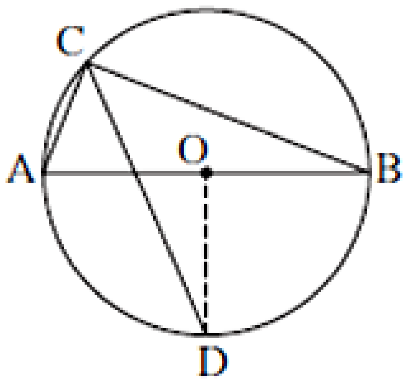


Fig. 3.98

Complete the following proof by filling in the blanks.
Proof: Draw seg OD.

$\angle ACB = \square$ ...90° .... angle inscribed in semicircle

$\angle DCB = \square$ ....45° .... CD is the bisector of $\angle C$

m(arc DB) = $\square$ ....45° .. inscribed angle theorem

$\angle DOB = \square$ ....90° definition of measure of an arc (I)

seg OA ≅seg OB ....radii of the circle... $\square$ (II)

∴line OD is $\square$ of seg AB ......bisector.... From (I) and (II)

∴seg AD ≅seg BD

Answer : Proof: Draw seg OD.

∠ ACB = 90° {angle inscribed in semicircle}

∠ DCB = 45° {CD is the bisector of $\sqrt{C}$}

m(arc DB) = 45° {inscribed angle theorem}

∠ DOB = 90° {definition of measure of an arc} (I)

seg OA ≅seg OB {radii of the circle}(II)

∴line OD is bisector of seg AB From (I) and (II)

∴seg AD ≅seg BD

Q. 21. In figure 3.99, seg MN is a chord of a circle with centre O. MN = 25,L is a point on chord MN such that ML = 9 and d(O,L) = 5.

Find the radius of the circle.

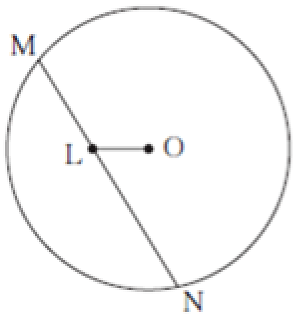
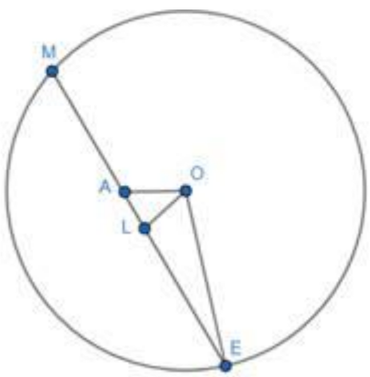


Fig. 3.99

Answer : The figure is shown below:



Draw perpendicular on MN from the center O.

Mark the point as A. Join O to N.

As we know that perpendicular on a chord bisects the chord.

AM = MN/2

⇒ AM = 25/2 = 12.5

Given that LM = 9

⇒ LM + LA = AM

⇒ 9 + LA = 12.5

⇒ LA = 3.5

In Δ OAL,

⇒ $OL^2 = OA^2 + AL^2$

⇒ $5^2 = OA^2 + (3.5)^2$

⇒ $OA^2 = 25-12.25$

⇒ $OA^2 = 12.75$

In Δ OAN,

⇒ $ON^2 = OA^2 + AN^2$

⇒ $ON^2 = 12.75 + (12.5)^2$

⇒ $ON^2 = 12.75 + 156.25$

⇒ $ON^2 = 169$

⇒ $ON = 13$

Therefore, the radius of the circle is 13.

Q. 22. In figure 3.100, two circles intersect each other at points S and R. Their common tangent PQ touches the circle at points P, Q.

Prove that, ∠ PRQ + ∠ PSQ = 180°

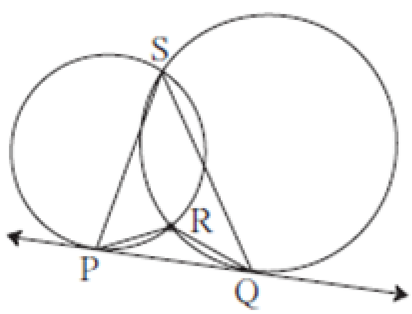


Fig. 3.100

Answer : We join R to S,

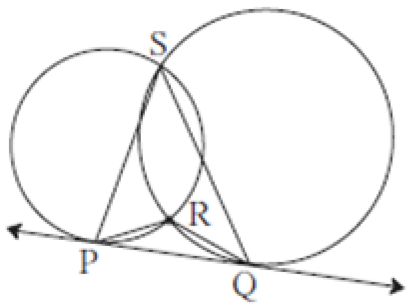


Fig. 3.100

As PQ is the tangent at P, we have

∠RPQ = ∠PSR …………..(1)

As PQ is tangent at Q, we have

∠RQP = ∠RSQ …………………(2)

In ΔRPQ, we have

⇒ ∠RPQ + ∠RQP + ∠ PRQ = 180° (Sum of all angles of a triangle)

⇒ ∠PSR + ∠RSQ + ∠PRQ = 180° (From (1) and (2))

⇒ ∠PSQ + ∠PRQ = 180° (∠PSR + ∠RSQ = ∠PSQ)

Hence Proved.

Q. 23. In figure 3.101, two circles intersect at points M and N. Secants drawn through M and N intersect the circles at points R, S and P, Q respectively.

Prove that :seg SQ || seg RP.

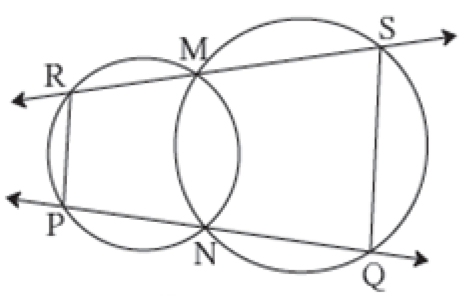
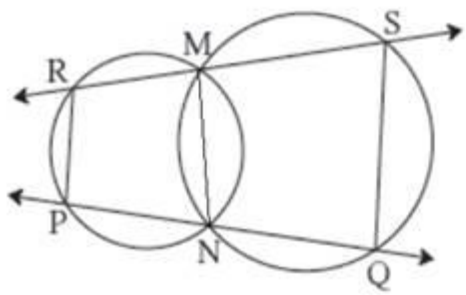


Fig. 3.101

Answer :



We join MN.

As PRMN is a cyclic quadrilateral,

$\angle R + \angle PNM = 180°$ …………………..(1) (opposite angles of a cyclic quadrilateral)

Also, QSMN is a cyclic quadrilateral,

$\angle S + \angle QNM = 180°$ ……………………(2) (opposite angles of a cyclic quadrilateral)

Adding (1) and (2)

$\angle R + \angle S + \angle PNM + \angle QNM = 360°$

$\Rightarrow \angle R + \angle S + 180 = 360$ (PQ is a straight line)

$\Rightarrow \angle R + \angle S = 180°$

Similarly we have,

As PRMN is a cyclic quadrilateral,

$\angle P + \angle RMN = 180°$ …………………..(3) (opposite angles of a cyclic quadrilateral)

Also, QSMN is a cyclic quadrilateral,

$\angle Q + \angle SMN = 180°$ ……………………(4) (opposite angles of a cyclic quadrilateral)

Adding (3) and (4)

$\angle P + \angle Q + \angle RMN + \angle SMN = 360°$

$\Rightarrow \angle P + \angle Q + 180 = 360$ (RS is a straight line)

$\Rightarrow \angle P + \angle Q = 180°$

Therefore, PR ∥ SQ.

Q. 24. In figure 3.102, two circles intersect each other at points A and E. Their common secant through E intersects the circles at points B and D. The tangents of the circles at points Band D intersect each other at point C.

Prove that □ ABCD is cyclic.

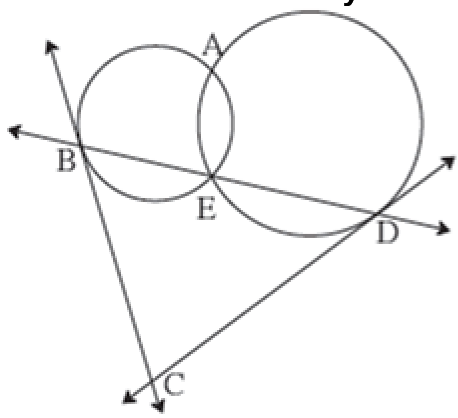
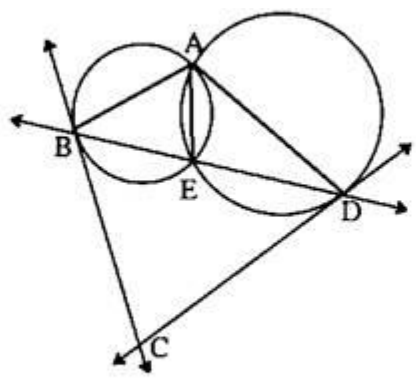


Fig. 3.102

Answer : We join A to B and A to D and A to E



As BC is a tangent at B, we have

∠ CBD = ∠BAE ………….(1)

As CD is a tangent at D, we have

∠ CDB = ∠DAE ………….(2)

In ΔBCD, we have

⇒ ∠CBD + ∠CDB + ∠ BCD = 180° (Sum of all angles of a triangle)

⇒ ∠BAE + ∠DAE + ∠BCD = 180° (From (1) and (2))

⇒ ∠BAD + ∠BCD = 180° (∠BAE + ∠DAE = ∠BAD)

In quadrilateral ABCD,

We have ∠A + ∠C = 180° (Proved above)

⇒ ∠A + ∠B + ∠C + ∠D = 360°

⇒ ∠B + ∠D + 180 = 360

⇒ ∠B + ∠D = 180

Therefore, opposite angles of the quadrilateral sum to 180. Hence ABCD is a cyclic quadrilateral.

Q. 25. In figure 3.103, seg AD ⊥ side BC, seg BE ⊥ side AC, seg CF ⊥ side AB. Point O is the orthocentre. Prove that, point O is the incentre of∆ DEF.

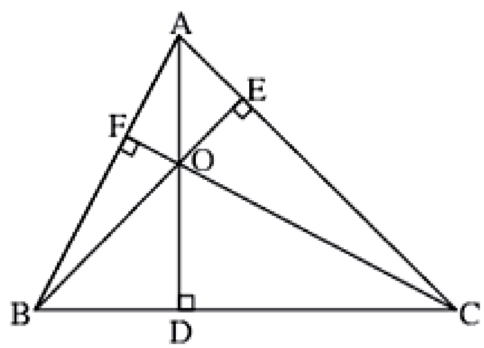
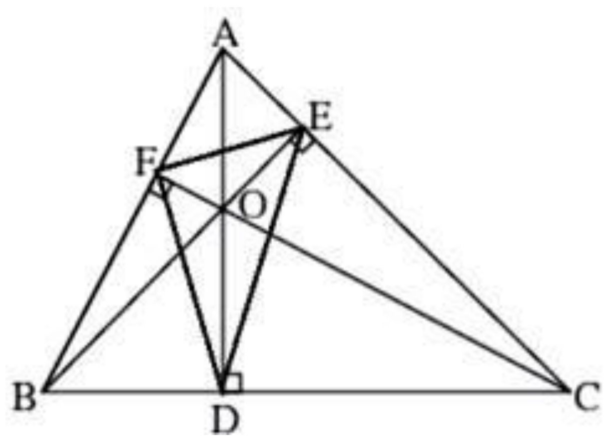


Fig. 3.103

Answer :



Join D to E, D to F and E to F.

In ΔABE,

$\Rightarrow \angle ABE + \angle BAE + \angle BEA = 180°$ (Sum of all angles of a triangle)

$\Rightarrow \angle ABE + \angle BAE + 90 = 180$

$\Rightarrow \angle ABE + \angle BAE = 90$

$\Rightarrow \angle ABO + \angle BAC = 90$

$\Rightarrow \angle ABO = 90 - \angle BAC$ ………………….(1)

In quadrilateral BFOD, we have

We have $\angle F = 90$, $\angle D = 90$

$\Rightarrow \angle B + \angle F + \angle O + \angle D = 360°$

$\Rightarrow \angle B + \angle O + 180 = 360$

$\Rightarrow \angle B + \angle O = 180$

Therefore, BFOD is a cyclic quadrilateral.

$\angle FBO = \angle FDO$ (angle by the same arc)

$\Rightarrow \angle ABO = \angle FDO$

From (1),

$\Rightarrow \angle FDO = 90 - \angle BAC$ ………………….(2)

In ΔAFC,

$\Rightarrow \angle CAF + \angle FCA + \angle AFC = 180°$ (Sum of all angles of a triangle)

$\Rightarrow \angle CAF + \angle FCA + 90 = 180$

$\Rightarrow \angle CAF + \angle FCA = 90$

$\Rightarrow \angle BAC + \angle OCE = 90$

$\Rightarrow \angle OCE = 90 - \angle BAC$ ………………….(3)

In quadrilateral CEOD, we have

We have $\angle E = 90$, $\angle D = 90$

$\Rightarrow \angle C + \angle E + \angle O + \angle D = 360°$

$\Rightarrow \angle C + \angle O + 180 = 360$

$\Rightarrow \angle C + \angle O = 180$

Therefore, CEOD is a cyclic quadrilateral.

$\angle ODE = \angle OCE$ (angle by the same arc)

From (3),

$\Rightarrow \angle ODE = 90 - \angle BAC$ …………(4)

From (2) and (4) we conclude,

$\angle FDO = \angle ODE$

OD bisects $\angle D$.

Similarly, we can prove that OE bisects $\angle E$ and OF bisects $\angle F$.

Hence O is the incenter of ΔDEF.