Trigonometry

Practice Set 6.1

Q. 1. If Sin$\theta = \frac{7}{25}$, find the values of cosθ and tanθ.

Answer : We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{7}{25}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{49}{625} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{49}{625}$

$\Rightarrow \cos^2\theta = \frac{576}{625}$

$\Rightarrow \cos\theta = \frac{24}{25}$

Also,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta = \frac{\left(\frac{7}{25}\right)}{\frac{24}{25}} = \frac{7}{24}$

Q. 2. If tan$\theta = \frac{3}{4}$, find the values of secθ and cosθ.

Answer : We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$$\Rightarrow \sec^2\theta = 1 + \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \sec^2\theta = 1 + \frac{9}{16}$$

$$\Rightarrow \sec^2\theta = \frac{25}{16}$$

$$\Rightarrow \sec\theta = \frac{5}{4}$$

Also,

$$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$$

$$\Rightarrow \cos\theta = \frac{4}{5}$$

Q. 3. If cot$\boldsymbol{\theta} = \frac{40}{9}$, find the values of cosecθ and sinθ.

Answer : We know that,

$\text{cosec}^2\theta = 1 + \cot^2\theta$

$$\Rightarrow \text{cosec}^2\theta = 1 + \left(\frac{40}{9}\right)^2$$

$$\Rightarrow \text{cosec}^2\theta = 1 + \frac{1600}{81}$$

$$\Rightarrow \text{cosec}^2\theta = \frac{1681}{81}$$

$$\Rightarrow Cosec\theta = \frac{41}{9}$$

Also,

$$\Rightarrow \sin\theta = \frac{1}{\text{cosec}\,\theta}$$

Q. 4. If 5secθ- 12cosecθ = 0, find the values of secθ, cosθ and sinθ.

Answer : 5secθ - 12cosecθ = 0

⇒ 5secθ = 12cosecθ

$$\Rightarrow \frac{\sec\theta}{\mathrm{cosec}\,\theta} = \frac{12}{5}$$

$$\Rightarrow \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \frac{12}{5}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{12}{5}$$

As $\tan\theta = \frac{\sin\theta}{\cos\theta}$ we have,

$$\Rightarrow \tan\theta = \frac{12}{5}$$

Also, We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$$\Rightarrow \sec^2\theta = 1 + \left(\frac{12}{5}\right)^2$$

$$\Rightarrow \sec^2\theta = 1 + \frac{144}{25}$$

$$\Rightarrow \sec^2\theta = \frac{169}{25}$$

$$\Rightarrow \sec\theta = \frac{13}{5}$$

Also,

$$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$$

$$\Rightarrow \cos\theta = \frac{5}{13}$$

Now, again using

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{12}{5} = \frac{\sin\theta}{\frac{5}{13}}$$

$$\Rightarrow \sin\theta = \frac{12}{5} \times \frac{5}{13}$$

$$\Rightarrow \sin\theta = \frac{12}{13}$$

Q. 5. If tanθ = 1 then, find the values of $\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta}$.

Answer : Given,

tan θ = 1

⇒ θ = 45° [as tan 45° = 1]

Also,

$$\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta}$$

$$= \frac{\sin\theta + \cos\theta}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$$

$$= \frac{\sin\theta + \cos\theta}{\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}}$$

= sinθ cosθ

$= \sin 45° \cos 45°$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

Q. 1. A. Prove that:

$$\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$$

Answer : Taking LHS

$$\frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$= \frac{1}{\cos\theta}$ [As, $\sin^2\theta + \cos^2\theta = 1$]

$= \sec\theta \left[\text{As}, \sec\theta = \frac{1}{\cos\theta}\right]$

= RHS

Proved !

Q. 6. B. Prove that:

$$\cos^2\theta\left(1 + \tan^2\theta\right) = 1$$

Answer : Taking LHS

$\cos^2\theta(1 + \tan^2\theta)$

$= \cos^2\theta \sec^2\theta$ [As, $\sec^2\theta = 1 + \tan^2\theta$]

$$= \cos^2\theta \times \frac{1}{\cos^2\theta} \left[\text{As}, \sec\theta = \frac{1}{\cos\theta}\right]$$

= 1

= RHS

Proved !

Q. 6.C. Prove that:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

Answer : Taking LHS

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times \frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}$$

$$= \frac{1-\sin\theta}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$ [(a + b)(a - b) = $a^2 - b^2$]

$$= \frac{1-\sin\theta}{\sqrt{\cos^2\theta}}$$ [As, $\sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

= RHS

Proved !

Q. 6. D. Prove that:

(sec θ – cos θ) (cot θ + tan θ) = tan θ sec θ

Answer : Taking LHS

(secθ - cosθ)(cotθ + tanθ)

= secθ cotθ + secθ tanθ – cosθ cotθ – cosθ tanθ

$$= \frac{1}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta}\right) + \sec\theta\tan\theta - \frac{\cos\theta\,(\cos\theta)}{\sin\theta} - \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} - \sin\theta + \tan\theta\sec\theta$$

Taking LCM of first three terms,

$$= \frac{1 - \cos^2\theta - \sin^2\theta}{\sin\theta} + \tan\theta\sec\theta$$

$$= \frac{1 - (\cos^2\theta + \sin^2\theta)}{\sin\theta} + \tan\theta\sec\theta$$

$= \frac{1-1}{\sin\theta} + \tan\theta\sec\theta$ [As, $\sin^2\theta + \cos^2\theta = 1$]

= tanθ secθ

= RHS

Proved !

Q. 6. E. Prove that:

cot θ + tan θ = cosec θ sec θ

Answer :

Taking LHS, and putting $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$ [As, $\sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta\sec\theta$$

= RHS

Proved !

Q. 6. F. Prove that:

$$\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$$

Answer : Taking LHS

$$= \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$= \sec\theta + \tan\theta$ [As, $\sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$]

= RHS

Proved !

Q. 6. G. Prove that:

$\sin^4 \theta - \cos^4 \theta = 1 - 2\cos^2 \theta$

Answer : L.H.S = $\sin^4\theta - \cos^4\theta$

= $(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

= $(\sin^2\theta - \cos^2\theta)$

= $(1 - \cos^2\theta - \cos^2\theta)$

= $1 - 2\cos^2\theta$

Q. 6. H. Prove that:

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1 - \sin\theta}$$

Answer : Taking RHS

$$= \frac{\cos\theta}{1 - \sin\theta}$$

$$= \frac{\cos\theta}{1 - \sin\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta}$$ (Multiplying both Numerator and Denominator by 1+SinΘ)

$$= \frac{\cos\theta(1 + \sin\theta)}{1 - \sin^2\theta}$$

$$= \frac{\cos\theta(1 + \sin\theta)}{\cos^2\theta}$$ [As, $\sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1 + \sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

= secθ + tanθ

= LHS

Proved !

Q. 6. I. Prove that:

If $\tan\theta + \frac{1}{\tan\theta} = 2$, then show that $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$

Answer : Given,

$$\left(\tan\theta + \frac{1}{\tan\theta}\right) = 2$$

Squaring both side,

$$\Rightarrow \tan^2\theta + \frac{1}{\tan^2\theta} + 2\tan\theta\left(\frac{1}{\tan\theta}\right) = 4 \ [(a+b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow \tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$$

$$\Rightarrow \tan^2\theta + \frac{1}{\tan^2\theta} = 2$$

Hence, Proved !

Q. 6. J. Prove that:

$$\frac{\tan A}{\left(1+\tan^2 A\right)^2} + \frac{\cot A}{\left(1+\cot^2 A\right)^2} = \sin A \cos A$$

Answer : Taking RHS

$$\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2}$$

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\mathrm{cosec}^2 A)^2}$$

$$= \frac{\tan A}{\sec^4 A} + \frac{\cot A}{\operatorname{cosec}^4 A}$$

$$= \frac{\sin A}{\cos A}.\cos^4 A + \frac{\cos A}{\sin A}.\sin^4 A$$

= $\sin A \cos^3 A + \cos A \sin^3 A$

= $\sin A \cos A(\cos^2 A + \sin^2 A)$ [As, $\sin^2\theta + \cos^2\theta = 1$]

= sinA cosA

= RHS

Proved !

Q. 6. K. Prove that:

$\sec^4 A\ (1 - \sin^4 A) - 2\tan^2 A = 1$

Answer : Taking LHS

= $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A$

= $\sec^4 A - \sin^4 A \sec^4 A - 2\tan^2 A$

$$= \sec^4 A - \frac{\sin^4 A}{\cos^4 A} - 2\tan^2 A$$

= $\sec^4 A - \tan^4 A - \tan^2 A - \tan^2 A$

= $\sec^4 A - \tan^2 A(1 + \tan^2 A) - \tan^2 A$

= $\sec^4 A - \tan^2 A \sec^2 A - \tan^2 A$ [As, $\sec^2\theta = 1 + \tan^2\theta$]

= $\sec^2 A(\sec^2 A - \tan^2 A) - \tan^2 A$

= $\sec^2 A - \tan^2 A$ [As, $\sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$]

= 1

= RHS

Proved !

Q. 6. L. Prove that:

$$\frac{\tan\theta}{\sec\theta-1}=\frac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}$$

Answer : Taking RHS

$$=\left(\frac{\tan\theta\ +\ \sec\theta\ +\ 1}{\tan\theta\ +\ \sec\theta-1}\right)\times\left(\frac{\sec\theta-1}{\sec\theta-1}\right)$$

$$=\frac{\tan\theta\sec\theta-\tan\theta\ +\ \sec^2\theta-\sec\theta\ +\ \sec\theta-1}{(\tan\theta\ +\ \sec\theta-1)(\sec\theta-1)}$$

$$=\frac{\tan\theta\sec\theta-\tan\theta\ +\ \sec^2\theta-1}{(\tan\theta\ +\ \sec\theta-1)(\sec\theta-1)}$$

$$=\frac{\tan\theta\sec\theta-\tan\theta+\tan^2\theta}{(\tan\theta+\sec\theta-1)(\sec\theta-1)}$$ [As $\sec^2\theta$ -1 = $\tan^2\theta$]

$$=\frac{\tan\theta\,(\tan\theta\ +\ \sec\theta-1)}{(\tan\theta\ +\ \sec\theta-1)(\sec\theta-1)}$$

$$=\frac{\tan\theta}{(\sec\theta-1)}$$

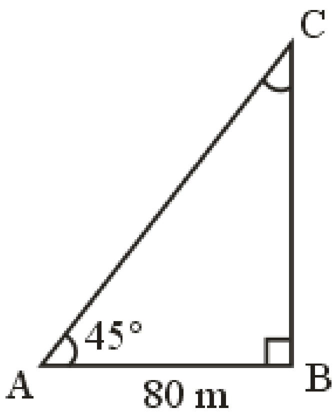
= LHS

Proved.

Practice Set 6.2

Q. 1. A person is standing at a distance of 80m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.

Answer :



Let 'A' be the person, standing 80 m away from a church BC,

Angle of elevation, ∠BAC = θ = 45°

Clearly, ΔABC is a right-angled triangle, in which

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\Rightarrow \tan 45° = \frac{BC}{80}$$

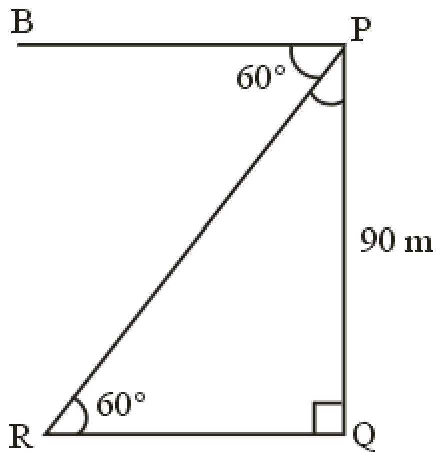
$$\Rightarrow 1 = \frac{BC}{80}$$

⇒ BC = 80 m

Therefore, Height of church is 80 m.

Q. 2. From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60°. If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse. (√3 = 1.73)

Answer :



Let PQ be a light house of height 80 cm such that PQ = 90 m

And R be a ship.

Angle of depression from P to ship R = ∠BPR = 60°

Also, ∠PRQ(say θ) = ∠BPR = 60° [Alternate Angles]

Clearly, PQR is a right-angled triangle.

Now, In ∆PQR

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PQ}{QR}$$

$$\Rightarrow \tan 60° = \frac{90}{QR}$$

$$\Rightarrow \sqrt{3} = \frac{90}{QR}$$

$$\Rightarrow QR = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3}$$

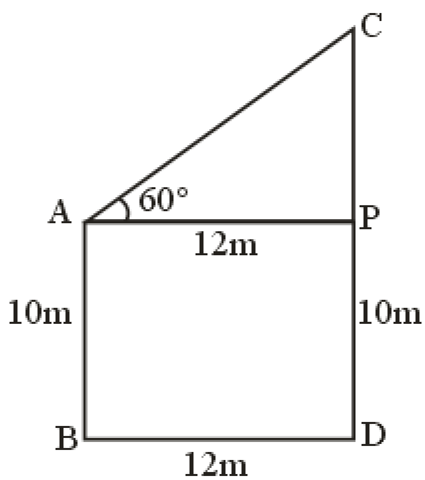
⇒ QR = 30(1.73)

⇒ QR = 51.90 m

Hence, Ship is 51.90 m away from the light house.

Q. 3. Two buildings are facing each other on a road of width 12 metre. From the top of the first building, which is 10 metre high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building?

Answer :



Let AB and CD be two building, with

AB = 10 m

And angle of elevation from top of AB to top of CD = $\angle CAP = 60°$

Width of road = BD = 12 m

Clearly, ABDP is a rectangle

With

AB = PD = 10 m

BD = AP = 12 m

And APC is a right-angled triangle, In ΔAPC

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CP}{AP}$$

$$\Rightarrow \tan 60° = \frac{CP}{12}$$

$$\Rightarrow \sqrt{3} = \frac{CP}{12}$$

⇒ CP = 12√3 m

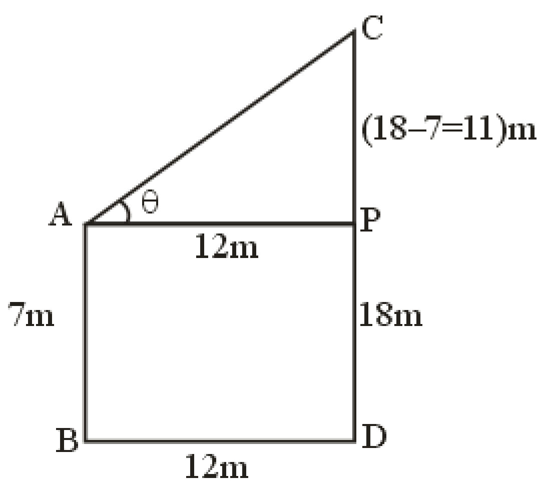
Also,

CD = CP + PD = (12√3 + 10) m

Hence, height of other building is (10 + 12√3 m).

Q. 4. Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops in 22 metre. Find the angle made by the wire with the horizontal.

Answer :



Let AB and CD be two poles, and AC be the wire joining their top.

And

Angle made by wire with horizontal = ∠CAP = θ [say]

Given,

AB = 7 m [Let AB be smaller pole]

CD = 18 m

AC = 22 m

Clearly, APDB is a rectangle with

AB = PD = 7 m

Also,

CP = CD – PD = 18 – 7 = 11 m

In ∆APC

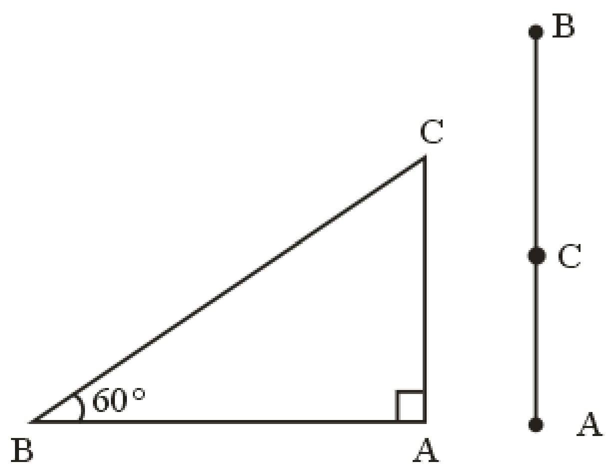
$$\sin\theta = \frac{CP}{AC}$$

$$\Rightarrow \sin\theta = \frac{11}{22} = \frac{1}{2}$$

⇒ θ = 30° $\left[\sin 30° = \frac{1}{2}\right]$

Q. 5. A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree.

Answer :



Let AB be a tree, and C is the point of break.

Height of tree = BC + AC

As, the treetop is rested 20 m from the base, making an angle of 60° with the horizontal.

In ∆ABC

AB = 20 m

∠ABC, θ = 60°

Now,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB}$$

$$\Rightarrow \tan 60° = \frac{AC}{20}$$

⇒ AC = 20 tan 60°

⇒ AC = 20√3 m

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{BC}$$

$$\Rightarrow \cos 60° = \frac{20}{BC}$$

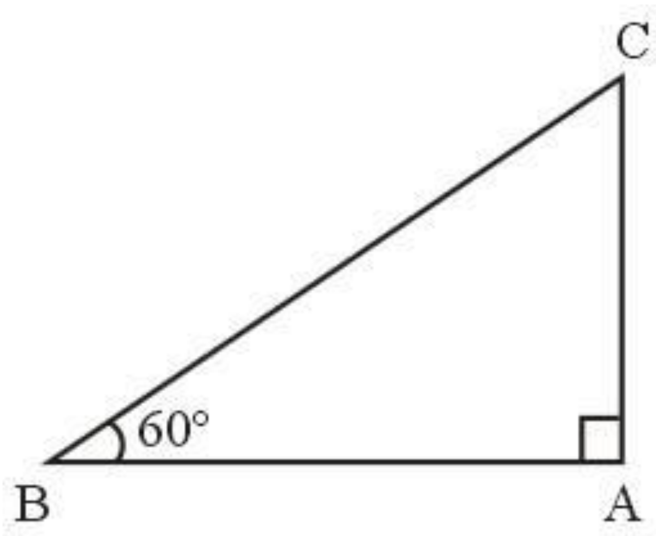
$$\Rightarrow \frac{1}{2} = \frac{20}{BC}$$

⇒ BC = 40 m

Height of tree = BC + AC = (40 + 20√3) meters.

Q. 6. A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. (√3 = 1.73)

Answer :



Let AC be a string and kite is flying at point A, with height

AB = 60 m

Angle make by string with horizontal, θ = ∠ACB = 60°

Clearly, ABC is a right-angled triangle.

In ∆ABC

$$\sin\theta = \frac{AB}{AC}$$

$$\Rightarrow \sin 60^\circ = \frac{60}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\Rightarrow AC = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$$

⇒ AC = 40(1.73)

⇒ AC = 6.92 m

Hence, length of string is 6.92 m

Problem Set 6

Q. 1. A. Choose the correct alternative answer for the following question.

sinθ cosecθ = ?

A. 1

B. 0

C. $\frac{1}{2}$

D. $\sqrt{2}$

Answer : We know,

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}$$

⇒ sinθ cosecθ = 1

Q. 1. B. Choose the correct alternative answer for the following question.

cosec45° =?

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer : As, cosec45° = √2

Q. 1. C. Choose the correct alternative answer for the following question.

$1 + \tan^2\theta = ?$

A. $\cot^2\theta$

B. $\text{cosec}^2\theta$

C. $\sec^2\theta$

D. $\tan^2\theta$

Answer : We know that,

$1 + \tan^2\theta = \sec^2\theta$

Q. 1. D. Choose the correct alternative answer for the following question.

When we see at a higher level, from the horizontal line, angle formed is.......
A. angle of elevation.
B. angle of depression.
C. 0
D. straight angle.

Answer : When we see at a higher level, from the horizontal line, the angle formed is known as angle of elevation.

Q. 2. If Sin$\theta = \frac{11}{6}$, find the values of cosθ using trigonometric identity.

Answer : As,

$\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{121}{3721} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{121}{3721} = \frac{3600}{3721}$$

$$\Rightarrow \cos\theta = \frac{60}{61}$$

Q. 3. If tan θ = 2, find the values of other trigonometric ratios.

Answer : We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta = 1 + (2)^2$

$\Rightarrow \sec^2\theta = 5$

$\Rightarrow \sec\theta = \sqrt{5}$ …[1]

Also,

$$\cos\theta = \frac{1}{\sec\theta}$$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$ …[2]

Now, using

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow 2 = \frac{\sin\theta}{\frac{1}{\sqrt{5}}}$$

$$\Rightarrow \sin\theta = 2 \times \frac{1}{\sqrt{5}}$$

$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$ ...[3]

Also,

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\sqrt{5}}{2}$ ...[4]

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2}$ ....[5]

Q. 4. If $\sec\theta = \frac{13}{12},$ **find the values of other trigonometric ratios.**

Answer : We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

$$\Rightarrow \tan^2\theta = \left(\frac{13}{12}\right)^2 - 1$$

$$\Rightarrow \tan^2\theta = \frac{169}{144} - 1 = \frac{25}{144}$$

$\Rightarrow \tan\theta = \frac{5}{12}$ … [1]

Also,

$$\cos\theta = \frac{1}{\sec\theta}$$

$\Rightarrow \cos\theta = \frac{12}{13}$ … [2]

Now, using

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{5}{12} = \frac{\sin\theta}{\frac{12}{13}}$$

$$\Rightarrow \sin\theta = \frac{5}{12} \times \frac{12}{13}$$

$\Rightarrow \sin\theta = \frac{5}{13}$ ... [3]

Also,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{13}{5}$ ... [4]

$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$ ....[5]

Q. 5. A. Prove the following.

secθ(1 - sinθ) (secθ + tanθ) = 1

Answer : Taking LHS

secθ(1 - sinθ) (secθ + tanθ)

$= \frac{1}{\cos\theta}(1 - \sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{\cos\theta}(1 - \sin\theta)\ \frac{1}{\cos\theta}(1 + \sin\theta)$

$= \frac{1}{\cos^2\theta}(1 - \sin^2\theta)$ [(a + b)(a - b) = $a^2 - b^2$]

$= \frac{1}{\cos^2\theta}(\cos^2\theta)$ [$\sin^2\theta + \cos^2\theta = 1$]

= 1

= RHS

Proved !

Q. 5. B. Prove the following.

(secθ + tanθ) (1 - sinθ) = cosθ

Answer : Taking LHS

(1 - sinθ) (secθ + tanθ)

$$= (1 - \sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= (1 - \sin\theta)\,\frac{1}{\cos\theta}(1 + \sin\theta)$$

$$= \frac{1}{\cos\theta}(1 - \sin^2\theta)\ [(a + b)(a - b) = a^2 - b^2]$$

$$= \frac{1}{\cos\theta}(\cos^2\theta)\ [\sin^2\theta + \cos^2\theta = 1]$$

= cos θ

= RHS

Proved !

Q. 5. C. Prove the following.

$\sec^2\theta + \mathrm{cosec}^2\theta = \sec^2\theta \times \mathrm{cosec}^2\theta$

Answer : Taking LHS

$\mathrm{Sec}^2\theta + \mathrm{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta\sin^2\theta}\ [\sin^2\theta + \cos^2\theta = 1]$$

$= \sec^2\theta \times \mathrm{cosec}^2\theta$

= RHS

Proved !

Q. 5. D. Prove the following.

$\cot^2\theta - \tan^2\theta = \text{cosec}^2\theta - \sec^2\theta$

Answer : Taking LHS

$\cot^2\theta - \tan^2\theta$

[Now, $\text{cosec}^2\theta - 1 = \cot^2\theta$ and $\sec^2\theta - 1 = \tan^2\theta$]

$= \text{cosec}^2\theta - 1 - (\sec^2\theta - 1)$

$= \text{cosec}^2\theta - \sec^2\theta$

= RHS

Proved !

Q. 5. E. Prove the following.

$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Answer : Taking LHS

$\tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$= (\sec^2\theta - 1)(\sec^2\theta)$ [$1 + \tan^2\theta = \sec^2\theta$]

$= \sec^4\theta - \sec^2\theta$

= RHS

Proved !

Q. 5. F. Prove the following.

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

Answer : Taking LHS

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$$

$= \frac{1 + \sin\theta + 1 - \sin\theta}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \frac{2}{1 - \sin^2\theta}$

$= \frac{2}{\cos^2\theta}$ [$\sin^2\theta + \cos^2\theta = 1$]

= 2 $\sec^2\theta$

= RHS

= Proved

Q. 5. G. Prove the following.

$\sec^6x - \tan^6x = 1 + 3\sec^2x \times \tan^2x$

Answer : Taking LHS

$\sec^6x - \tan^6x$

$= (\sec^2x)^3 - (\tan^2x)^3$

$= (\sec^2x - \tan^2x)(\sec^4x + \tan^2x \sec^2x + \tan^4x)$

[As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= \sec^4x + \tan^4x + \tan^2x \sec^2x + 2\tan^2x \sec^2x - 2\tan^2x \sec^2x$

[As, $\sec^2\theta - \tan^2\theta = 1$]

$= \sec^4x + \tan^4x - 2\tan^2x \sec^2x + 3\tan^2x \sec^2x$

$= (\sec^2x - \tan^2x)^2 + 3\tan^2x \sec^2x$ [$a^2 + b^2 - 2ab = (a - b)^2$]

$= 1^2 + 3\tan^2x \sec^2x$

$= 1 + 3\tan^2x \sec^2x$

= RHS

Proved.

Q. 5. H. Prove the following.

$$\frac{\tan\theta}{\sec\theta+1}=\frac{\sec\theta-1}{\tan\theta}$$

Answer : Taking LHS

$$=\frac{\tan\theta}{\sec\theta\ +\ 1}\times\frac{\sec\theta-1}{\sec\theta-1}$$

$$=\frac{\tan\theta\,(\sec\theta-1)}{\sec^2\theta-1}$$

$$=\frac{\tan\theta(\sec\theta-1)}{\tan^2\theta}$$ [$\tan^2\theta = \sec^2\theta - 1$]

$$=\frac{\sec\theta-1}{\tan\theta}$$

= RHS

Proved !

Q. 5. I. Prove the following.

$$\frac{\tan^3\theta-1}{\tan\theta-1}=\sec^2\theta+\tan\theta$$

Answer : Taking LHS

$$\frac{\tan^3\theta-1}{\tan\theta-1}$$

$$=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta-1}$$ [$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

= $\tan^2\theta + \tan\theta + 1$

= $\sec^2\theta + \tan\theta$ [$1 + \tan^2\theta = \sec^2\theta$]

= RHS

Proved.

Q. 5. J. Prove the following.

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$

Answer : Taking LHS

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing numerator and denominator by cosθ

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$$= \frac{(\tan\theta - 1 + \sec\theta)(\sec\theta - \tan\theta)}{(\tan\theta + 1 - \sec\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{\tan\theta\sec\theta - \tan^2\theta - \sec\theta + \tan\theta + \sec^2\theta - \tan\theta\sec\theta}{(\tan\theta + 1 - \sec\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{\tan\theta + 1 - \sec\theta + \sec^2\theta - \tan^2\theta}{(\tan\theta + 1 - \sec\theta)(\sec\theta - \tan\theta)}$$

$= \frac{\tan\theta + 1 - \sec\theta}{(\tan\theta + 1 - \sec\theta)(\sec\theta - \tan\theta)}$ [As $\sec^2\theta - \tan^2\theta = 1$]

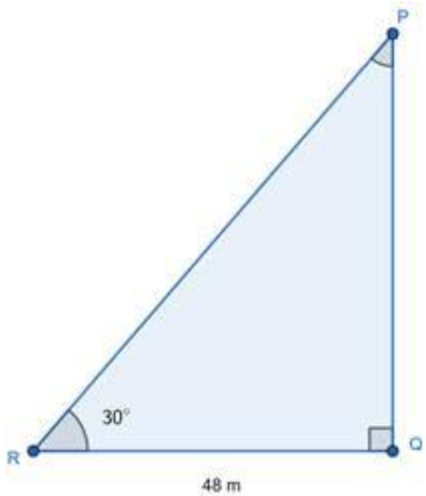
$$= \frac{1}{\sec\theta - \tan\theta}$$

= RHS

Proved.

Q. 6. A boy standing at a distance of 48 meters from a building observes the top of the building and makes an angle of elevation of 30°. Find the height of the building.

Answer :



Let 'R' be the person, standing 48 m away from a building PQ,

Angle of elevation, ∠PRQ = θ = 30°

Clearly, ∆ABC is a right-angled triangle, in which

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PQ}{QR}$$

$$\Rightarrow \tan 30° = \frac{BC}{48}$$

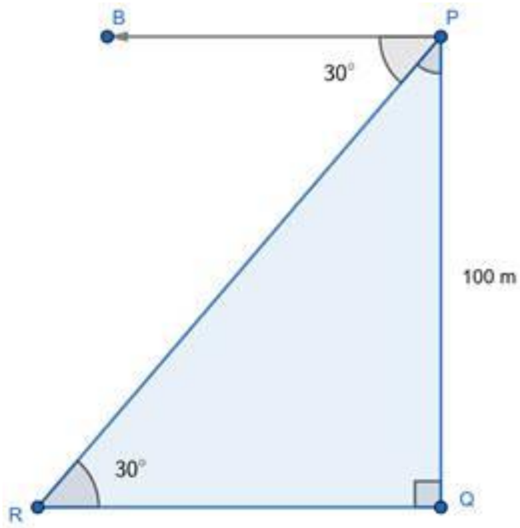
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{48}$$

$$\Rightarrow BC = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3} = 16\sqrt{3}\text{ m}$$

Therefore, Height of church is 16√3 m.

Q. 7. From the top of the light house, an observer looks at a ship and finds the angle of depression to be 30°. If the height of the light-house is 100 meters, then find how far the ship is from the light-house.

Answer :



Let PQ be a light house of height 80 cm such that PQ = 100 m

And R be a ship.

Angle of depression from P to ship R = ∠BPR = 30°

Also, ∠PRQ(say θ) = ∠BPR = 30° [Alternate Angles]

Clearly, PQR is a right-angled triangle.

Now, In ∆PQR

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PQ}{QR}$$

$$\Rightarrow \tan 30° = \frac{100}{QR}$$

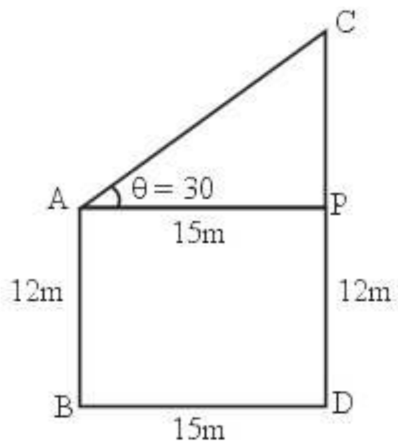
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{QR}$$

⇒ QR = 100√3 m

Hence, Ship is 100√3 m away from the light house.

Q. 8. Two buildings are in front of each other on a road of width 15 meters. From the top of the first building, having a height of 12 meter, the angle of elevation of the top of the second building is 30°.What is the height of the second building?

Answer :



Let AB and CD be two building, with

AB = 12 m

And angle of elevation from top of AB to top of CD = ∠CAP = 30°

Width of road = BD = 15 m

Clearly, ABDP is a rectangle

With

AB = PD = 12 m

BD = AP = 15 m

And APC is a right-angled triangle, In ΔAPC

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CP}{AP}$$

$$\Rightarrow \tan 30° = \frac{CP}{15}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CP}{15}$$

$$\Rightarrow CP = \frac{15}{\sqrt{3}} = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$$

⇒ CP = 5√3 m

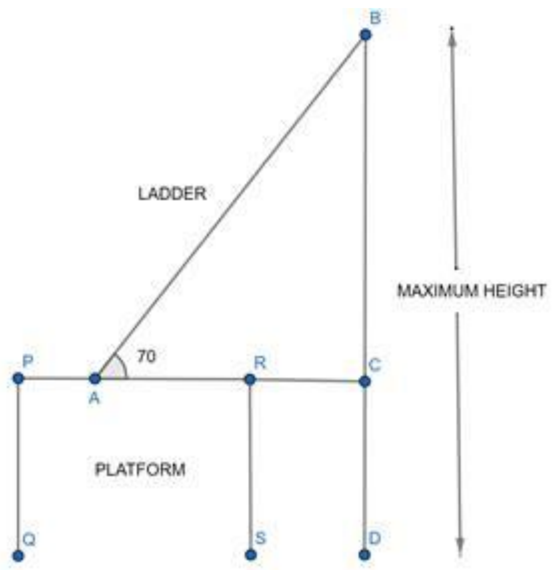
Also,

CD = CP + PD = (5√3 + 12) m

Hence, height of other building is (12 + 5√3 m).

Q. 9. A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20m. If the platform is 2m above the ground, find the maximum height from the ground upto which the ladder can reach. (sin 70° = 0.94)

Answer :



Let AB be the ladder, i.e. AB = 20 m and PQRS be the platform.

In the above figure, clearly

PQ = RS = CD = height of platform = 2 m

Maximum height ladder can reach = BC + CD

Now,

Maximum value of ∠BAC = 70°

In right-angled triangle ABC,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$\Rightarrow \sin 70° = \frac{BC}{20}$$

$$\Rightarrow 0.94 = \frac{BC}{20}$$

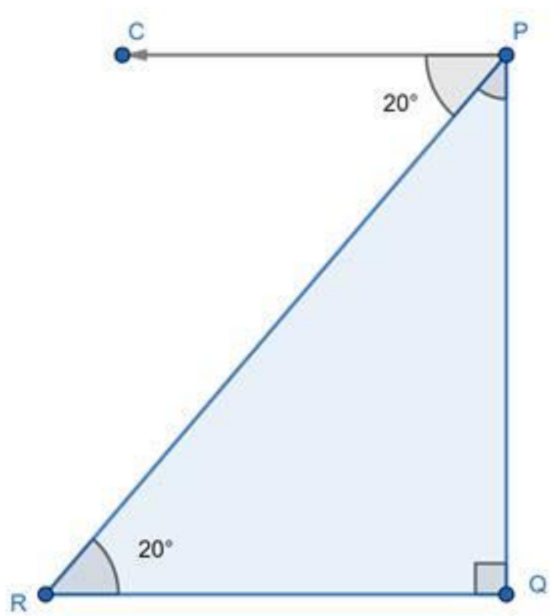
⇒ BC = 18.8 m

Maximum height ladder can reach = BC + CD

= 18.8 + 2 = 20.8 meters

Q. 10. While landing at an airport, a pilot made an angle of depression of 20°. Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing. (Sin 20° = 0.342)

Answer :



Let the initial position of plain be P and after landing it position be R.

Now,

Angle of depression while landing, ∠CPR = 20°

Also, ∠ CPR = ∠PRQ (say θ) = 20° [Alternate Angles]

Now,

Speed of plane = 200 km / hr

$$= 200 \times \frac{1000}{3600} = \frac{500}{9}\ \mathrm{ms}^{-1}$$

Time taken for landing = 54 seconds

Distance travelled in landing = PR

Also, distance = speed × time

$$\Rightarrow \mathrm{PR} = \frac{500}{9} \times 54 = 3000\ \mathrm{m}$$

Now, In ∆PQR

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\mathrm{PQ}}{\mathrm{PR}}$$

$$\Rightarrow \sin 20° = \frac{\mathrm{PQ}}{3000}$$

⇒ PQ = 3000 × sin 20°

⇒ PQ = 3000(0.342)

⇒ PQ = 1026 m

So, Plane was at a height of 1026 m at the start of landing.